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Sustainability of a fiscal policy and a current account: A threshold cointegration approach for the G-7 countries

by

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1 Introduction

During the last decade the importance of a sustainable fiscal policy and current account has been increasingly in the scope of economists and policy makers. In principle, an economy will be able to sustain deficits, weather fiscal or external, as long as it can raise the necessary funds by borrowing. Although such behavior may be feasible in the short run, the ability of the economy to service its debt by resorting to further borrowing is likely to be questioned once the deficits become persistent.

In the case of a public deficit, authorities traditionally justify moderate fiscal deficits by pointing to the need to avoid an excessive build-up of debt and the resulting pressure on monetary policy. The need for funds to fill the fiscal holes may imply a constant pressure on the domestic capital markets. This will result in higher domestic interest rates and, in turn, high costs in foregone output and employment. Sustained deficits may also cause a higher inflation rate, even if the deficit is not financed directly by printing money.

This last concern was discussed by Sargent and Wallace (1985); they model an economy with a persistent deficit financed by treasury issued bonds while the monetary authority maintains a passive policy. In time, the real interest obligations of the treasury would rise making it possible that the revenue from new bond sales would be insufficient to pay the service on past bonds. When this rollover option fails, the government is forced to issue money to pay off the deficit. Sargent and Wallace showed that this increased interest expense may necessitate faster money growth in the future, yielding higher inflation today.

Given the detrimental impact of persistent deficit finance practices on debt accumulation, inflation rates, interest rates and economic growth, answering the crucial question of whether current fiscal deficits can be sustained in the long-run is of particular interest for economists and policy makers.

The growing concern over the last two decades regarding unsustainable fiscal policies has helped many industrialized economies make their transition from chronic deficits to moderate deficits or even surplus. For example, the EU countries committed to fiscal discipline with the Maastricht Treaty, and until recently the U.S. was expected to run surpluses for the first years of the century. On the other hand developing countries are still fighting to balance their budgets; for example, Latin American governments like those in Argentina, Colombia, Mexico, and Brazil are still carrying sizable deficits.

The resurgence of interest in the issue of the external imbalances has arisen from two developments in the world economy: the widespread debt servicing problems of many developing countries, and the large and persistent current account imbalances of the three main industrial countries. The large payments' imbalances of the United States, Germany, and Japan have been accompanied by dramatic changes in their foreign indebtedness. For example, the United States was the largest net creditor country for much of the post-World War II period but now is one of the world's largest debtors. Germany and Japan in return have accumulated large net foreign claims.

Short-run disequilibria in the current account may not be considered bad, since they may reflect reallocation of capital from one country to another. These disequilibria may be simply explained by the capital looking for a more productive country. But persistent payments' imbalances are a cause for both domestic and international concern primarily because of the undesirable consequences of a sharp 'forced' adjustment by the private or public sector if such tendencies are expected to continue. To sustain an increasing current account deficit implies measures such as increasing interest rates to attract foreign capital. This measure imposes an excessive burden on future generations, thus lowering future standards of living.

Once the debtor country is unable to borrow to cover the current account deficits, it will be forced to take actions such as reducing public deficits and stimulating private savings to correct persistent current account deficits. The sooner both lender and borrower are aware of the true situation the better. For this reason, it is important to be able to assess whether present paths of external imbalance can indefinitely continue conditional on present policies being maintained.

As argued before, the sustainability of a current fiscal policy and a current account is a key issue in the viability of an economy. Throughout this paper an unsustainable situation will be defined broadly as one in which economic variables cannot continue indefinitely on their historical paths as implied by current policies and private sector behavior. In this situation, the economy is not on a long-run steady-state path and some policies must be changed in the future.

According to this definition, an unsustainable situation is not a crisis situation. A fiscal or current account crisis is a state of affairs where the system is close or has collapse, so that a change in the economic structures has to be made. On the other hand an unsustainable situation is one situation when changes are needed to bring back the economy to the long-run sustainable path, but those changes do not need to be instantly. Of course, an unsustainable situation is the path to a crisis, but they are not the same. In that sense, this study will not study fiscal or current account crises. The theoretical and empirical analysis presented here are intended to detect sustainable or unsustainable situations, some steps down from an actual crisis.

In this dissertation we consider the traditional theoretical tools available to detect an unsustainable fiscal policy or current account. We will argue that a relatively new econometric concept currently available may be useful to reconsider the sustainability problem. In doing so, the concept, tests procedures, and estimation techniques involved in the notion of threshold cointegration are reviewed. Then these new tools will be applied to evaluate the sustainability of the fiscal policies and current accounts of the G-7 countries (Canada, France, Germany, Italy, Japan, the U.K. and the U.S.).

This dissertation is organized as follows. The first chapter corresponds to this briefly introduction. The second chapter is dedicated to review and classify the theoretical and empirical work relevant to the sustainability of a fiscal policy and a current account. The third chapter, discusses the concept of threshold cointegration and summarizes the most popular cointegration test available to identify linear and non-linear cointegration. Chapter 3 also presents a Monte Carlo study designed to study the power of those cointegration tests. Chapter 3 finalizes with a description of the estimation and testing techniques available for estimating a Threshold Vector Error Correction Model. In Chapter 4 we study the sustainability of the fiscal policy and current account of the G-7 countries. The last chapter, Chapter 5, summarizes our findings and suggests new research.

2 Sustainability of a Fiscal Policy and a Current Account: Literature Review

This chapter discusses the pertinent literature on sustainability of both the fiscal deficit and the current account. The chapter is divided into two main parts. The first part examines the literature relevant to the long-run solvency of the public sector and the second part summarizes literature about the external solvency of the economy.

2.1 Sustainability of a Fiscal Policy

The sustainability of a fiscal policy, understood as the possibility of preserving current expenditure and tax structures without any eventual change in the long run, has been studied formally and empirically for nearly two decades. This area of study began with the seminal paper by Hamilton and Flavin (1986) that provided the basis for tackling this problem. Since then, the theoretical tools available to answer the sustainability question have not changed, but the econometric tools have.

In the literature on fiscal policy sustainability there have been basically two main approaches. Both approaches suggest possible techniques to test the sustainability of a fiscal policy via the fulfillment of the government's Present Value Borrowing Constraint –PVBC–. One approach, beginning with Hamilton and Flavin (1986), is based on the univariate time series properties of the government debt; the second approach has its origins in Hakkio and Rush's (1991) research and is based on the long-run relationship between total fiscal revenues and expenditures —bivariate approach.

This section presents a literature review of the studies in this area. The discussion is organized into four parts. The first part describes the common ground for the two different approaches in this field. The second part studies the univariate approach and the third section studies the bivariate approach. The last section discusses other work related to the field that cannot be classified in the previous two sections. A table summarizing the empirical results from all the reviewed papers is provided at the end of the section –Table 2.1.

2.1.1 The Basic Model.

Both approaches to the study of fiscal sustainability commence with the budget restriction that a government faces each period. The government revenues[1] and the newly issued debt should cover the purchases of goods and services and the service of the debt:

$$G_t + r_t B_{t-1} = R_t + B_t - B_{t-1}, \tag{2.1}$$

where G_t are expenses net of debt service, R_t are revenues, B_t is the stock of public debt, and r_t is the interest rate in period t. Reordering terms in (2.1):

$$G_t + (1 + r_t) B_{t-1} = R_t + B_t . \tag{2.2}$$

This budget restriction is satisfied in period t, as well as in the subsequent periods. Thus, recursive forward substitution on equation (2.2) yields:

$$B_t = \sum_{i=1}^{\infty} \delta_i (R_{t+i} - G_{t+i}) + \lim_{i \to \infty} \delta_{t+i} B_{t+i} , \tag{2.3}$$

where $\delta_i = \prod_{s=1}^{i} 1/(1 + r_{t+s})$.

Many authors have pointed out that expression (2.3) is not controversial and has no economic interest, since it comes from an accounting identity[2]. The interesting feature is the expectation of the public regarding the value of the second term on the right-hand-side of (2.3). If creditors expect to be repaid and thus the government is able to keep its current expenditure-tax structure, then the value of current debt should be equal to the present value of all the future non-interest surpluses. In other words, the Present Value Borrowing Constraint (PVBC) faced by the government implies that the fiscal authority is not able to

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[1] The government revenues include all the receipts, i.e., including seignorage.

[2] For example: Hamilton and Flavin (1986), Wilcox (89), Hakkio and Rush (1991).

use a Ponzi scheme to finance its deficit. Formally,

$$B_t = \sum_{i=1}^{\infty} E_t\left[\delta_i \left(R_{t+i} - G_{t+i}\right)\right], \tag{2.4}$$

where $E_t[\bullet]$ represents the expectation of the public based on the information available at period t. Note that this implies that the government can run a permanent non-interest deficit without being in jeopardy of violating the PVBC. Also note that an excessive stock of debt has to imply pressures over the capital market and thus over the interest rate. In a crisis situation, there are not funds available to the fiscal authority at any interest rate, and prior to or during a crisis situation high interest rates will reflect this crisis. Since we are interested t in investigating a sustainable or unsustainable situation rather than a crisis, where the problem has been blown out of proportions, we will consider a case were the effect of the building debt has not reached "critical" levels, so that the sustainability problem is not big enough to have a sizable effect on the interest rate[3].

Different authors have tried to investigate the sustainability of fiscal policy testing the null hypothesis that (2.4) holds or equivalently:

$$E_t\left[\lim_{i \to \infty} \delta_{t+i} B_{t+i}\right] = 0. \tag{2.5}$$

Equation (2.5) is also known as the transversality condition. The difference in the two main approaches is how to test (2.5). Testing (2.5) is not an easy task, since it pertains to expectations which are unobservable. Different assumptions and interpretations of the above test are described in the next two sections.

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[3] This will be reflected in assuming a constant or stationary interest rate.

2.1.2 Univariate Approach

Hamilton and Flavin (1986) ask themselves whether the government faces a borrowing constraint such as the one faced by any individual. Assuming a constant interest rate and accounting for any short-run error in the expectations $-\varepsilon_t-$, (2.3), the PVBC becomes

$$B_t = E_t \sum_{i=1}^{\infty} \frac{(R_{t+i} - G_{t+i})}{(1+r)^i} + (1+r)^t A_o + \varepsilon_t, \tag{2.6}$$

where $A_o = \frac{\lim_{i\to\infty} B_{t+i}}{(1+r)^i}$. Hamilton and Flavin proposed three tests for $A_o = 0$ using U.S. annual data for the period 1960-1984.

First, they observed that B_t is stationary if and only if the process for the discounted sum of future surpluses is stationary and $A_o = 0$. To implement this test, they performed a Dickey-Fuller test for unit roots on debt and surpluses. They found both series to be stationary and thus concluded $A_o = 0$, i.e., the U.S. government was satisfying its PVBC.

The second test proposed by Hamilton and Flavin (1986) was based on the direct estimation of (2.6) under the assumption that expectations are based in part on past values of the surplus. Thus, they estimate different versions of the following equation,

$$B_t = A_o(1+r)^t + c(L)(R_t - G_t) + d(L)B_{t-1}. \tag{2.7}$$

Again, they found the estimate for A_o not significantly different from zero.

For the last test, they assume that expectations are formed only by lagged values of the surplus and estimated jointly a pair of equations representing debt as a function of expected future surpluses and the surpluses as a function of its own past values. Again, the estimate for A_o is not significantly different from zero. Thus, all three tests gave Hamilton and Flavin (1986) enough evidence to conclude that the government was not violating the intertemporal budget constraint.

Wilcox (1989), assuming a stochastic real interest rate, suggested that an unsustainable fiscal policy could be detected examining the forecast trajectory for the present discounted value of B_t. If the forecast trajectory converges to zero under the current policy, then the policy is sustainable.

Wilcox (1989), using the same data used by Hamilton and Flavin (1986), found no evidence of non-stationarity of the discounted debt and estimated an AR(2) and an AR(3) model for the discounted debt. For both cases, the null of an unconditional mean equal to zero is rejected. Wilcox did not stop there. He performed a stability test –Chow test– on the parameters of the AR(2) model by dividing the sample into two periods: 1962-74 and 1974-84. He found evidence of coefficient instability, re-estimated the model, and discovered evidence to reject the null of an unconditional mean equal to zero for the period 1974-84. Thus, he concluded that the U.S. fiscal policy for the period 1974-1984 was not sustainable in contrast with the sustainable fiscal policy maintained during the 1962-74 period.

Trehan and Walsh (1991) developed a theoretical framework to test the sustainability of the fiscal balances as well as the external balances without assuming a constant or stationary interest rate. In particular they showed that the stationarity of the inclusive-of-interest deficit is necessary and sufficient for an inter-temporal budget balance, when the expected real interest rate is constant and the fiscal balance is I(1). Using the same data set used by Hamilton and Flavin – 1960-84 – they found evidence in favor of the sustainability of the fiscal policy of the U.S.

There have been numerous studies using these kinds of models for different countries. Smith and Zin (1991), closely following Hamilton and Flavin (1986), studied the sustainability of the Canadian fiscal policy using monthly data for the period 1946-1984. They used a conventional DF test to test for stationarity of the primary surplus and the debt for different subsamples. They found that the Canadian fiscal policy was not sustainable, and that the result was robust to different subsamples.

Banglioni and Cherubini (1994) found evidence in favor of the unsustainability of the Italian fiscal policy for monthly data (Jan1979-91). Baffes and Shah (1994) found sustainable fiscal policies for Mexico (Annual data: 1895-1984) and Argentina (Annual data: 1913-84), and discovered evidence of an unsustainable fiscal policy for Brazil (Annual data: 1907-1985).

Balfoussias, et al. (1999) recognized the potential problem of using conventional unit root tests to assert the stationarity of a series with the presence of policy regime shifts –shifts in the mean or trend. They used the Kwiatkowski, et al. (1992) unit root test to test the stationarity of the discounted debt to GDP ratio with an unknown change in the trend[4]. They provide evidence in favor of the sustainability of the Turkish fiscal policy.

Feve, et al. (1998) recognized, as suggested by Hansen (1995), that unit root tests to test for sustainability, such as the one suggested by Wilcox (19991), have low power. Sustainability tests often ignore the joint dynamics of stock (debt) and flow variables – deficit–. With Monte Carlo simulations, they show that a conventional unit root test leads to incorrect statistical inference, and that taking into account the joint dynamics when testing for unit roots induces large power gains. They introduced a Feedback Augmented Dickey Fuller that combines the Covariate Augmented Dickey-Fuller test proposed by Hansen (1995) with feedback effects outlined by the auxiliary regression approach to test for sustainability (for example Wickens and Uctum (1993a)).

Feve, et al. (1998) use U.S. annual data for the period 1792 - 1988 to test the stationarity of the discounted debt. They found that their approach was robust for different discount factors in comparison with traditional unit root tests. They found evidence in favor of the

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[4] This test modifies the Perron (1989) testing procedure by considering a random walk process with drift that excludes any structural change under the null. The alternative hypothesis is a trend stationary process with a one-time break in the trend, but the time of the break is taken as unknown. The breakpoint selection procedure is done by finding the breakpoint that produces the lowest value of the relevant one-sided unit root t-statistic.

stationarity of the debt, i.e., the U.S. fiscal policy is consistent with the PVBC.

Feve and Henin (2000) apply the Feedback Augmented Dickey Fuller (FADF) test to test sustainability for the fiscal policies of G7 countries. This test is applied to semi-annual public debt normalized by GDP (a Hamilton and Flavin type of test). The null hypothesis of non-sustainability still cannot be rejected for Germany, France, Italy and Canada.

2.1.3 Bivariate Approach

Hakkio and Rush's (1991) approach departs from the univariate approach by assuming that the interest rate is stationary with mean r [5]. They add and subtract rB_{t-1} to both sides of equation (2.1) to obtain:

$$G_t + (r_t - r)B_{t-1} + (1+r)B_{t-1} = R_t + B_t. \tag{2.8}$$

Forward iterating on (2.8), the Intertemporal Budget Constraint is given by

$$B_t = \sum_{i=1}^{\infty} \mu_i [R_t - G_t] + \lim_{i\to\infty} \mu_i B_{t+i}, \tag{2.9}$$

where $\mu_i = \prod_{j=1}^{i} \left(\frac{1}{1+r_{t+j}} \right)$, i.e. the discount factor.

Note that equation (2.9) implies that when the limiting term tends to zero, the current value of the debt has to be equal to the present discount value of future government primary surpluses and deficits. If the limiting term does not go to zero, then the government is "bubble" financing its expenditure, i.e., old debt is paid only by issuing new debt (Ponzi scheme). Therefore, a fiscal policy will be sustainable[6] if the limiting term is zero.

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[5] Assuming that the interest rate is stationary will imply that the testable relationship cannot be estimated in nominal terms, since the stationarity of the nominal interest rate is questionable.

[6] i.e., the PVBC is satisfied.

Upon further manipulation, equation (2.9) can be written as[7]:

$$GG_t - R_t = \sum_{i=0}^{\infty} \frac{\Delta R_{t+i} - \Delta GG_{t+i} + r\Delta B_{t+i-1}}{(1+r)^{i-1}} + \lim_{i\to\infty} \frac{B_{t+i}}{(1+r)^{i+1}}, \tag{2.10}$$

where GG_t represents the government expenditure in goods and services as well as the service of the debt, i.e., $GG_t = G_t + r_t B_{t-1}$.

Hakkio and Rush (1991) assume that R_t and $G_t + (1+r)B_{t-1}$ are random walks with drift:

$$R_t = \alpha_1 + R_{t-1} + \varepsilon_{1t} \tag{2.11}$$

and

$$GG_t = \alpha_2 + GG_{t-1} + \varepsilon_{2t}. \tag{2.12}$$

Consequently, (2.10) can be rewritten as

$$GG_t = \alpha + R_t + \lim_{i\to\infty} \frac{B_{t+i}}{(1+r)^{i+1}} + \varepsilon_t, \tag{2.13}$$

where $\alpha = \frac{1+r}{r}(\alpha_1 - \alpha_2)$ and $\varepsilon_t = \sum_{i=0}^{\infty} \frac{(\varepsilon_{1t} - \varepsilon_{2t})}{(1+r)^{i-1}}$.

Now, they have achieved a testable expression. If the transversality condition holds the third term in the right-hand side of (2.13) has to be zero. Thus, if transversality holds, we can rewrite (2.13) as a regression equation:

$$R_t = a + bGG_t + \varepsilon_t, \tag{2.14}$$

along with the null hypothesis $b = 1$ and ε_t a stationary process. In other words, if the PVBC is satisfied, and if R and GG are not stationary, then they have to be cointegrated with cointegrating vector [1, -1].

To see why $b = 1$ and cointegration between R and GG implies $\lim_{i\to\infty} B_{t+i}/(1+r)^{i+1} = 0$, suppose without lost of generality that $r_t = r$ for all t. Substituting $\hat{a} + \hat{b}GG_t$ for R_t in (2.2),

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[7] **For intermediate steps see Hakkio and Rush (1991) p.432.**

the following expression is found:

$$G_t + (1+r)B_{t-1} = \left(\hat{a} + \hat{b}GG_t\right) + B_t$$

$$G_t + (1+r)B_{t-1} = \left(\hat{a} + \hat{b}(G_t + rB_{t-1})\right) + B_t$$

$$B_t = G_t\left(1-\hat{b}\right) - \hat{a} + \left(1 + r\left(1-\hat{b}\right)\right)B_{t-1}. \tag{2.15}$$

Forward iterating on (2.15), any B_{t+i} can be written as:

$$B_{t+i} = \sum_{k=0}^{i}\left[1+r\left(1-\hat{b}\right)\right]^{i-k} S_{t+k} + \left[1+r\left(1-\hat{b}\right)\right]^{i} B_{t-1}. \tag{2.16}$$

Now replacing (2.16) into the limiting term of equation (2.13):

$$\lim_{i\to\infty}\frac{B_{t+i}}{(1+r)^{i+1}} = \lim_{i\to\infty}\left[\sum_{k=0}^{i}\frac{\left[1+r\left(1-\hat{b}\right)\right]^{i-k}}{(1+r)^{i+1}}S_{t+k} + \frac{\left[1+r\left(1-\hat{b}\right)\right]^{i}}{(1+r)^{i+1}}B_{t-1}\right] \tag{2.17}$$

where $S_t = \left(1-\hat{b}\right)G_t - \hat{a}$, in other word spending. Note that the limiting term will be zero if $\hat{b} = 1$, since (2.17) becomes $\lim_{i\to\infty}\left[\sum_{k=0}^{i}\frac{1}{(1+r)^{i+1}}S_{t+k} + \frac{1}{(1+r)^{i+1}}B_{t-1}\right] = 0$ for $\hat{b} = 1$.

When normalizing the expenses and the revenues by the GDP, the PVBC still holds if $0 < b \leq 1$. Hakkio and Rush point out that although $0 < b \leq 1$ is consistent with a "strict" interpretation of the intertemporal budget restriction, this condition is inconsistent with the requirement that the ratio debt/GDP is finite. If $0 < b < 1$ (and the expenses and revenues are expressed as a percentage of the GDP), the real value of the debt as a percentage of the GDP diverges to infinity[8]. In sum, the existence of cointegration between the expenses and the tax revenues is a necessary condition for the PVBC to hold. It is not a sufficient condition,

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[8] It is also important to note that the limiting term in (2.13) will be also equal to zero if $0 < b < 1$, since $1 + r\left(1-\hat{b}\right) < (1+r)$ and thus the denominator of the limiting terms grows faster than the numerator. However, if $0 < b < 1$, then the limit of the undiscounted value of the debt is infinite – $\lim_{i\to\infty} B_{t+i} =$ $\lim_{i\to\infty}\sum_{k=0}^{i}\left[1+r\left(1-\hat{b}\right)\right]^{i-k} S_{t+k} + \lim_{i\to\infty}\sum_{k=0}^{i}\left[1+r\left(1-\hat{b}\right)\right]^{i-k} B_{t-1} = \infty$. Therefore we require that $b = 1$ in order to have sustainability.

whereas $b=1$ is "probably" a necessary condition (Hakkio and Rush (1991)).

Hakkio and Rush recommended testing the sustainability of the fiscal policy using transformed series, series as a ratio of the GDP and series in real terms. They did the exercise using quarterly data for 3 nested samples. First they tested for stationarity of the series with the DF test and then tested for cointegration using the Durbin-Watson statistic, DF/ADF statistic, the Stock and Watson statistic and the restricted and unrestricted VAR statistic. For the period 1950:2–1988:4 the series appear to be cointegrated but with the parameter b less than one. For the periods 1964:1–1988:4 and 1978:1–1988:4, the series are not cointegrated. Thus, they conclude the U.S. fiscal policy was not sustainable for that period.

Haug (1995) extends the work of Hakkio and Rush by introducing a semi-parametric test of parameter stability to find out whether a structural break occurs or not. He found that government deficit policy in the 1980s was not significantly different from policies during the three earlier decades. However, a diverging debt-GNP ratio suggests that the government will run into problems marketing its debt if current policy continues.

Quintos (1995) extended the work of Hakkio and Rush by discussing the condition for deficit sustainability and searches for shifts in the structure of U.S. deficit policy. The author showed that cointegration between revenues and expenditures inclusive of debt payment is not a necessary but a sufficient condition for a strict interpretation of deficit sustainability. The necessary and sufficient condition is that debt grows slower than the borrowing rate. Using tests that search for shifts in the rank of the cointegrating matrix, the author shows that the deficit is sustainable despite the failure of cointegration in the 1980s.

Payne (1997), following Hakkio and Rush, tested for the sustainability of the fiscal policies in the G-7 countries. He used an ADF test to test the non-stationarity and cointegration of the series. Payne studied the natural logarithm of the annual series in real terms, as a ratio of the GDP, and in per-capita terms. He found a sustainable fiscal policy only for the German case.

2.1.4 Other Approaches

The studies reviewed in this section differ from the framework developed before. For example, Bohn (1995) used a general equilibrium model for a stochastic dynamically efficient economy with risk averse individuals to derive the intertemporal constraints for the fiscal policy. He found that the discounting of future government debt, revenues and expenditure in the intertemporal budget constraint and the transversality condition generally depends on the probability distribution of these variables. In contrast to the literature reviewed above, the constraints cannot be written in terms of expected fiscal variables discounted at a fixed interest rate, except in special cases.

Ahmed and Rogers (1995) extension of Hakkio and Rush (1991) is to consider a stochastic environment. The authors departed from the fact, proven by Bohn (1995), that the sequence of period-by-period government budget constraints – as expressed by (2.1) – and the Euler equation from the consumer's optimization problem $-E_t\left[(1+r_t)MRS_{t+1}^t\right]=1$, where MRS_{t+i}^t is the marginal rate of substitution between consumption in period t and $t+i$– give the following equation:

$$E_t\left[\sum_{i=0}^{\infty} MRS_{t+i}^t G_{t+i}\right]-E_t\left[\sum_{i=0}^{\infty} MRS_{t+i}^t R_{t+i}\right]+(1+r_{t-1})B_{t-1}=\lim_{i\to\infty} E_t\left(MRS_{t+i}^t G_{t+i}\right) \quad (2.18)$$

Note that (2.18) is exactly the standard Intertemporal Budget Constraint –as expression (2.9) –, except that the discounting factor is the marginal rate of substitution.

Upon further algebra manipulation, (2.18) can be written as

$$\begin{aligned}\Delta E_t\left[\sum_{i=0}^{\infty} MRS_{t+i}^t G_{t+i}\right]-\Delta E_t\left[\sum_{i=0}^{\infty} MRS_{t+i}^t R_{t+i}\right]+(G_t+r_t B_{t-1}-R_t)\\ =\lim_{i\to\infty} E_t\left(MRS_{t+i}^t B_{t+i}\right)-\lim_{i\to\infty} E_{t-1}\left(MRS_{t+i-1}^{t-1} B_{t+i-1}\right)\end{aligned} . \quad (2.19)$$

Ahmed and Rogers, demonstrated that a necessary condition for sustainability is that the right-hand-side of (2.19) is equal to zero –and tax revenues, expenditures, and debt service have to be cointegrated with cointegrating vector (1, -1, -1). Furthermore, the authors showed that assuming that government expenditure and tax revenues are I(1) and government debt has the following behavior: $B_t = \mu + B_{t-1} + \lambda^t + u_t$ [9], then the same cointegration relationship is a sufficient condition for sustainability. Thus, a test for the fulfillment of the government's present value constraint consists of estimating the following cointegrating equation

$$R_t = \beta_o + \beta_1 G_t + \beta_2 r_t B_{t-1} \tag{2.20}$$

and testing the null hypothesis that $\beta_1 = \beta_2 = 1$, i.e., the government's fiscal policy is sustainable[10].

Ahmed and Rogers (1995), using annual data for the United States and the United Kingdom for the periods 1889-1992 and 1830-1992 respectively, found that the present value constraints hold over the whole sample period. The data also indicate that the present value constraints continue to hold following events that cause a structural break in the short-run dynamics.

Using another framework, Bohn (1998) investigated a systematic relationship between the debt-income ratio and the primary surplus, and estimated a regression of the form,

$$R_t - G_t = \rho B_t + \alpha Z_t + \varepsilon_t = \rho B_t + \mu_t \tag{2.21}$$

where Z_t is a set of other determinants of the primary surplus, ε_t is an error term, and $\mu_t = \alpha Z_t + \varepsilon_t$.

Bohn (1998) showed that an estimated positive response of primary surpluses to the debt-GDP ratio can be interpreted as a new test for the sustainability of U.S. fiscal policy. It provides strong evidence that U.S. fiscal policy was sustainable in the sense of the PVBC for the sample period 1916–1995 and various subperiods.

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[9] Where μ and λ are constants and u_t is a zero-mean stationary process.

[10] Note that this is equivalent to Hakkio and Rush's test In other words, Hakkio and Rush's Test is robust to a stochastic environment.

Table 2.1. Summary of Empirical Results. Sustainability of a Fiscal Policy

Paper	Variables Used	Country	Data			Method	Sust?
			Freq.	from	to		
Banglioni and Cherubini (1993)	Debt P. Surplus	ITA	M	Jan-79	May-91	Uni	No
Baffes and Shah (1994)	R, R/GDP G, G/GDP	ARG	A	1913	1984	Uni	Yes
		BRA		1907	1985		No
		MEX		1895	1984		Yes
Haug (1995)	R, R/GDP, R/N G, G/GDP,G/N	U.S.	Q	1950:II	1988:IV	Bi	Yes
Quintos (1995)	R, R/GDP, R/N G, G/GDP,G/N	U.S.	A	1950	1993	Bi	Yes
Ahmed and Rogers (1995)	Debt P. Surplus	U.S.	A	1692	1994	O	Yes
		U.K.					Yes
Payne (1997)	R, R/GDP, R/N	G-7	A				
	G, G/GDP,G/N	CAN		1949	1993	Bi	No
	(in logs)	FRA		1950	1993		No
		GER		1951	1993		Yes
		ITA		1951	1993		No
		JAP		1955	1993		No
		U.K.		1949	1993		No
		U.S.		1949	1994		No
Bohn (1998)	D/GDP, R/GDP G/GDP	U.S.	A	1916	1995	O	Yes
Feve, et al. (1998)	D /GDP, R/GDP G/GDP	U.S.	A	1792	1988	Uni	Yes
Balfoussias, et.al (1999)	D / GDP	GRE	A	1958	1999	Uni	Yes
Feve and Henin (2000)	D /GDP,	G-7	S-A				
		CAN				Uni	No
		FRA		50 data points			No
		GER		75 data points			No
		ITA		50-75 data pts			No
		JAP		50 data points			Yes
		U.K.		50-75 data pts			Yes
		U.S.		75 data points			Yes

Notes:1) Freq and Meth stand for frequency of the data used and method used in the paper, respectively.
2) The column "Sust?" represents the question Is the fiscal policy sustainable?
3) A, M, Q and S-A stand for annual, monthly, quarterly and semiannual data, respectively.
4) Uni, Bi, and O represent a univariate method, bivariate method, and other method, respectively.

Table 2.1. (Cont.) Summary of Empirical Results. Sustainability of a Fiscal Policy

Paper	Variables Used	Country	Freq.	Data from	to	Meth.	Sust?
Hamilton and Flavin (1986)	Debt P. Surplus	U.S.	A	1960	1984	Uni	Yes
Wilcoxon (1989)	R, G and D	U.S.	A	1960	1974	Uni	Yes
				1974	1984		No
Trehan and Walsh (1991)	Debt P. Surplus	U.S.	A	1960	1984	Uni	Yes
Haug (1991)	R G	U.S.	A	1890	1986	Bi	Yes
Smith and Zin (1991)	Debt P. Surplus	CAN	M	1946	1984	Uni	No
Hakkio and Rush (1991)	R, R/GDP, R/N	U.S.	Q	1950:II	1988:IV	Bi	No
	G, G/GDP,G/N			1964:II	1988:IV		No
				1976:II	1988:IV		No

Notes:1) Freq and Meth stand for frequency of the data used and method used in the paper, respectively.

2) The column "Sust?" represents the question Is the fiscal policy sustainable?

3) A, M, Q and S-A stand for annual, monthly, quarterly and semiannual data, respectively.

4) Uni, Bi, and O represent a univariate method, bivariate method, and other method, respectively.

2.2 Sustainability of a Current Account

Traditionally, the literature on the current account has suggested different ways to assess if an external deficit is so big as to imply a future change in policies. Several measures of potential national insolvency have been proposed, for example:

- *The proportion of foreign net worth held in a particular* country's *debt*[11].

  This is an ad hoc criterion associated with the demand side of asset holding and is not likely to be a good guide to the composition of international portfolios which tend to be determined by the conventional financial considerations of risk and return.
- *The ratio of foreign indebtedness to domestic GNP* (Krugman (1989)).

  This measure is an attempt to capture supply side effects and is used to indicate the ability of a country to service its debt. It is, however, a rather informal way of doing so.
- *The real rate of interest on national debt adjusted for output and population growth* (Cohen (1985), (1988) and Viñals (1986)).

  A value less than zero implies that running a zero mean trade balance will result in the ratio of foreign indebtedness to domestic GNP converging to zero and hence long-run current account balance will be sustainable for any level of initial indebtedness.

In the 1990's the literature on the current account related to the measures of potential national insolvency began to introduce new econometric ideas such as cointegration and non-stationarity in the mix. Based on developments in the literature on the sustainability of the government deficit, Husted (1992) originated a series of studies on the sustainability of the current account deficit based on the time series properties of the relevant macroeconomic aggregates. Since then the development of the literature on sustainability of current account and of the fiscal policy have experienced a parallel development due to the similar theoretical nature of the problems.

As in the case of the sustainability of fiscal policy, the theoretical arsenal has not changed too much, but the empirical tools available have. This section presents a literature review on research in national external insolvency. The discussion is organized in four parts. The first part briefly describes the similarities of the problem of a sustainable current account and a sustainable fiscal policy. The second section describes the univariate approach. The third section studies the bivariate approach. The last section discusses other work related with the field that cannot be classified in the previous two sections. A table summarizing the empirical results is provided at the end of the section – Table 2.2..

2.2.1 Similarities between the Problem of the Sustainability of a Current Account and a Fiscal Policy

The nature of the sustainability of a current account problem is the same as the sustainability of a fiscal deficit. In principle, in both problems we have for each period a stock (debt) that is enlarged by a positive flow (deficit) or attenuated by a negative flow (surplus). Thus, our question is whether the stock is becoming so large that we cannot "pour" more into the stock, and thus the behavior of the system has to be changed.

In the case of a fiscal policy, $G_t + r_t B_{t-1} - R_t$ is the flow that will increase or decrease the stock of public debt. In the case of a current account, $CA_t = M_t - X_t + r_t B^f_{t-1}$ (where M_t is imports in period t, X_t is exports in period t, B^f_t is the stock of one-period foreign debt issued in period t, and r_t is the one-period world interest rate) is the flow that enlarges or abridges the stock of foreign debt. Therefore, the analysis of current account sustainability is clearly very similar in nature to the analysis of a sustainable fiscal policy.

In section 2.1.1 we recognized that the sustainability of a public policy implies the

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[11] e.g. Isard and Stekler (1985).

transversality condition (2.5), i.e. $E_t\left[\lim_{i\to\infty}\delta_{t+i}B_{t+i}\right]=0$. For the case of a current account, given the similar nature of the two problems, sustainability implies an analogous condition, i.e. $E_t\left[\lim_{i\to\infty}\delta_{t+i}B^f_{t+i}\right]=0$. The fact that this conditional expectation is unobservable leads researchers to use different approaches to test the fulfillment of the transversality condition. As in the sustainability of a fiscal policy, this gives origin to two main approaches to test the transversality condition: a univariate and a multivariate. In the next two sections a detailed review of the different approaches to the problem is reported.

2.2.2 Univariate Approach

One of the most common univariate approaches is taken by Liu and Tanner (1996). This approach starts from the per-period budget constraint faced by the country expressed in real terms:

$$M_t - X_t + r_t B^f_{t-1} = \Delta B^f_{t-1} \quad . \tag{2.22}$$

Assuming that the interest rate is stationary with mean r ($r_t = r + \upsilon_t$, with υ_t a zero-mean random error), forward iteration of (2.22) gives

$$B^f_t = \sum_{i=1}^{\infty}\frac{M_{t+1+i} - X_{t+1+i}}{(1+r)^{i+1}} + \lim_{i\to\infty}\frac{B^f_{t+1+i}}{(1+r)^{i+1}} + \sum_{i=1}^{\infty}\frac{v_{t+1+i}}{(1+r)^{i+1}} \, . \tag{2.23}$$

Now, assuming that the exports and imports series are I(1)[12] and taking expected values, (2.23) may be written as

$$CA_t = \theta + \lim_{i\to\infty} E_t\left[\frac{B^f_{t+k+1}}{(1+r)^{k+1}}\right] + \omega_t , \tag{2.24}$$

where ω_t is a stationary error term[13] and θ is a constant[14]. From our discussion in the

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[12] $M_t = \mu_1 + M_{t-1} + \varepsilon_{1,t}$ and $X_t = \mu_2 + X_{t-1} + \varepsilon_{2,t}$.

[13] This error term is a function of V_t, $\varepsilon_{1,t}$, and $\varepsilon_{2,t}$.

previous section, it is clear that the current account is sustainable if the second term in the right-hand-side is zero. Thus if the current account is sustainable, then the current account series has to be a stationary process.

Liu and Tanner (1996) used (2.24) to test the external solvency of France, Germany, Italy, Japan, Canada, the United Kingdom, and the United States. The quarterly data used in the study corresponded to the period beginning in the early 1970's and finishing in the early 1990's, but for each country the data set has a different length. The authors utilized a Dickey-Fuller test with a break in the intercept to account for shifts in the behavior of the series in the 1980's. They concluded that U.S., Germany, and Japan fulfilled the requirements for a sustainable current account.

As mentioned before, Trehan and Walsh (1991) developed a univariate test for the sustainability of the current account (for details see section 2.1.2) using annual data for the foreign debt during the period 1946-87. They found that the current account balance was sustainable for the U.S. economy during that period.

2.2.3 Bivariate Approach

Husted (1992) was the first author to use cointegration techniques to test the sustainability of the current account. He developed a theoretical framework to test for sustainability based on Hakkio and Rush's (1991) procedure.

Husted's approach began by noting that an open economy faces the following budget constraint for each period t

$$C_t = Y_t + B_t^f - I_t - (1+r_t)B_t^f, \tag{2.25}$$

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[14] $\theta = \dfrac{(\mu_1 - \mu_2)(1+r)}{r}$

where C_t is the public and private consumption in period t, Y_t is the production in period t, and I_t is investment in period t.

Since this budget constraint must be satisfied for all periods, forward iterating (2.25), the intertemporal budget constraint is given by

$$B_t^f = \sum_{i=1}^{\infty} \mu_i \left[Y_{t+i} - C_{t+i} - I_{t+i}\right] + \lim_{i \to \infty} \mu_i B_i^f , \tag{2.26}$$

where $\mu_t = \prod_{j=1}^{i} \left(\frac{1}{1+r_{t+j}} \right)$ is the product of the first i discount factors. Note that

$$Y_t - C_t - I_t = X_t - M_t = TB_t \tag{2.27}$$

where TB denotes trade balance.

Therefore the economy's budget constraint can be expressed as

$$B_t^f = \sum_{i=1}^{\infty} \mu_i \left[TB_{t+i}\right] + \lim_{i \to \infty} \mu_i B_i^f . \tag{2.28}$$

Equation (2.28), like the government's intertemporal budget constraint, says that when the last term vanishes the current value of the foreign debt has to be equal to the sum of present discounted value of future trade balances. If, for example, the current stock of foreign debt is bigger than the present value of future trade balances, then the country's debt is in a "bubble" and thus the current account is not sustainable.

Following Hakkio and Rush (1991), Husted (1992) assumed a stationary world interest rate with mean r that is exogenous with respect to this economy's choices. Thus, as in Hakkio and Rush (1991)[15], expression (2.28) can be written as

$$M_t + rB_{t-1}^f = X_t + \sum_{i=0}^{\infty} \frac{\Delta X_{t+i} - \Delta Z_{t+i}}{(1+r)^{i-1}} + \lim_{i \to \infty} \frac{B_{t+i}^f}{(1+r)^{i-1}} , \tag{2.29}$$

where $Z_t = M_t + (r_t - r)B_{t-1}^f$. Now, subtracting X_t and then multiplying both sides of the later

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[15] See previous section.

equation by minus 1, we get

$$CA_t = X_t - M_t - rB^f_{t-1} = \sum_{i=0}^{\infty} \frac{\Delta Z_{t+i} - \Delta X_{t+i}}{(1+r)^{i-1}} - \lim_{i\to\infty} \frac{B^f_{t+i}}{(1+r)^{i-1}}. \tag{2.30}$$

Again following Hakkio and Rush (1991), Husted assumed that X and Z are I(1) processes given by

$$X_t = \alpha_1 + X_{t-1} + \varepsilon_{1t} \tag{2.31}$$

$$Z_t = \alpha_2 + Z_{t-1} + \varepsilon_{2t} \tag{2.32}$$

with ε_{it} stationary processes.

For this particular case, equation (2.30) becomes

$$X_t = \alpha + MM_t - \lim_{i\to\infty} \frac{B^f_{t+i}}{(1+r)^{i+1}} + \varepsilon_t, \tag{2.33}$$

with $MM_t = M_t - r_t B^f_{t-1}$, $\alpha = \frac{(1+r)}{r}(\alpha_1 - \alpha_2)$, and $\varepsilon_t = \sum_{i=0}^{\infty} \frac{(\varepsilon_{1t} - \varepsilon_{2t})}{(1+r)^{i-1}}$.

Assuming that the second term in (2.33) vanishes, then (2.33) can be written as a simple regression relation

$$X_t = a + b \cdot MM_t + \varepsilon_t \tag{2.34}$$

Thus, under the null hypothesis of sustainability of the current account we expect that $b = 1$ and ε_t will be stationary. In other words, as shown by Hakkio and Rush, if X_t and MM_t are I(1) and measured relative to GDP[16], then under the null they are cointegrated with cointegrating vector [1, -*1*].

With the previous theoretical framework, Husted (1992) tested the sustainability of the U.S. current account using quarterly data for the period 1967:1–1989 4 and the Engle and Granger cointegration test. He found that the current account deficit for that period was not

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[16] See section 2.1.3.

sustainable. But he realized that it seems there might have been a structural break after 1983:4 in the cointegrating relationship between exports and imports. Thus, he included a dummy variable in the cointegration equation to capture the break and performed an Engle and Granger test with new Monte Carlo-simulated critical values. He found that the current account was sustainable and that the long-run tendency of this account's balance had shifted from zero to $100 billion per year (Real Dollars of 1985).

Wu, Fountas and Chen (1996), following Husted (1992), test the sustainability of the current account for Canada and United States. Using quarterly data for the period 1974-1994, they found that the series are not cointegrated. "To allow for possible changes in the cointegrating vector over the estimation period,"[17] they decided to test the stationarity of the current account deficit with Zivot and Andrews' (1992) unit root test. With this univariate test they found evidence in favor of the sustainability of the current account for both countries.

Note that the univariate test is imposing the restriction that the cointegrating vector is (1,-1) for the whole period. The only thing that the Zivot and Andrews' test is doing is to test whether or not the intercept of the cointegration errors is the same during the period. This test does not allow for changes in the cointegrating vector as claimed by the authors and thus is not a valid test for sustainability according to the theoretical framework they used.

Apergis, et al. (2000), again following Husted (1991), test for the sustainability of the Greek current account with annual data for the period 1960–1994. They use Gregory and Hansen's (1996) cointegration method that allows to test for cointegration under regime-shifts. They also use Stock and Watson's (1993) method to estimate the cointegrating vector that includes a deterministic component. They found that the Greek current account deficit was sustainable.

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[17] Wu, Fountas and Chen (1996), page 195.

2.2.4 Other Approaches

Home (1991) developed a sustainability index based on the idea that unsustainable policies will induce changes in the behavior of the private sector that have to be incorporated at the moment of forecasting the future behavior of the current account deficit. He suggested the calculation of conditional forecasts to access the sustainability of the present current account balances.

Wickens and Uctum (1993b), unlike the models in the bivariate approach, do not assume an exogenous primary deficit and thus they develop a complete macroeconomic model. They derive a two-equation VAR in which the trade deficit becomes an endogenous variable and derive conditions for the fulfillment of the economy's intertemporal budget constraint. They found for the case of United States during the period 1970:1–1988:4 that the current account was sustainable.

Ahmed and Rogers (1995) not only proposed a test for the sustainability of the fiscal policy in a stochastic environment, but also suggested one for the sustainability of the current account and the economy as a whole. Using the economy's external per-period budget constraint, equation (2.22) , assuming that M_t and X_t are I(1) processes, and following exactly the same procedure reported in the previous section, Ahmed and Rogers (1995) found that a sufficient condition for external solvency is that the following cointegration relation is met

$$X_t = \phi_0 + \phi_1 M_t + \phi_2 r_{t-1} B^f_{t-1}, \tag{2.35}$$

with cointegrating vector [1,-1,-1].

Ahmed and Rogers (1995), proposed an "Economic-Wide Balance" test to see if the economy is jointly satisfying the government and external intertemporal budget constraints. This "Economic-Wide Balance" relationship appears when households internalize both the government and external present value constraints. We can get the "Economic-Wide Balance" constraint substituting (2.20) and (2.35) into $Y_t = C_t + I_t + G_t + X_t - M_t$. The authors, using annual data for the United States and United Kingdom for the periods 1889-1992 and 1830-1992, respectively, found that the present value constraints hold over the whole sample period.

Leachman and Francis (2000) used a multicointegration analysis for a U.S. quarterly data set over the period 1947:I–1994:VI. They split the sample into two subsamples comprising the Bretton Woods and post-Bretton Woods periods, i.e. before and after 1974. The authors found that during the Bretton Woods period imports and exports shared two long run equilibrium relationships that tied them together in such a way that the external budget was consistent with a sustainable current account process. On the other hand, for the post-Bretton Woods period, imports and exports did not exhibit any long run equilibrium relationship. Thus the authors concluded that the U.S. was engaged in an unsustainable current account situation.

Table 2. Summary of Empirical Results. Sustainability of a Current Account

Paper	Variables Used	Country	Data			Meth.	Sust?
			Freq.	from	to		
Trehan and Walsh (1991)	Foreign Debt	U.S.	A	1946	1987	Uni	Yes
Husted (1992)	X/GDP, M/GDP	U.S.	Q	1967:I	1989:VI	Bi	No
Wickens and Uctum (1993)		U.S.	Q	1970:I	1988:VI	O	Yes
Wu, Fountas, and Chei (1996)	X/GDP, M/GDP	U.S.	Q	1974:VI	1994:VI	Bi	Yes
		CAN					Yes
						Uni	Yes
							Yes
Apergis et al. (2000)	X/GDP, M/GDP	GRE	A	1960	1994	Bi	Yes
Liu and Tanner (1996)	Current Account	FRA	Q	1970	1990	Uni	No
		GER	Q	1970	1990		Yes
		ITA	Q	1970	1990		No
		JAP	Q	1970	1990		Yes
		CAN	Q	1970	1990		No
		U.K.	Q	1970	1990		No
		U.S.	Q	1970	1990		Yes
Leachman and Francis (2000)	X/GDP, M/GDP Current Account Foreign Debt	U.S.	Q	1947:I	1994:IV	O	Yes (1947-1974) No (1975-1994)

Notes: 1) Freq and Meth stand for frequency of the data used and method used in the paper, respectively.
2) The column "Sust?" represents the question: Is the current account sustainable?
3) A, M, and Q stand for annual, monthly, and quarterly data, respectively. Uni. Bi, and O represent a univariate method, bivariate method, and other method, respectively.

3 Testing Cointegration with Non-Linear Adjustment Processes and Estimating Threshold Vector Error Correction Models

Since the introduction of the concept of cointegration by Engle and Granger (1987), the use of cointegration tests has been a key tool used by economists to study the long-run relationship between economic variables. At the same time, economists have turned their attention to the use of non-linear time series techniques to model univariate economic series[18].

However, Balk and Fomby (1997) were the first authors to introduce the possibility of a nonlinear long-run relationship among economic variables. Their work has become a converging point for literature about cointegration and non-linear time series. Before considering the details involved in non-linear cointegration systems, let's consider the linear cointegration case.

In general, the time series in the vector $x_t = (x_{1,t}, x_{2,t}, \ldots, x_{n,t})$ are cointegrated of order s, g – CI(s,g) – if all the components of x_t are integrated of the same order s – I(s) – and there exists a vector $\beta = (\beta_1, \beta_2, \ldots, \beta_n)$ such that the linear combination $\beta x_t = z_t$ is integrated of order s-g. Thus, if, for example, we have a random vector x_t with all its components being integrated of order one, then the elements of x_t are said to be cointegrated if there exists a linear combination such that $z_t = \beta x_t$ is I(0)[19]. In the rest of the paper we will only consider the latter case.

The interpretation behind a cointegration relation between non-stationary variables is very appealing for economists. Even though, in the short-run, each individual series seems to be erratically moving, if cointegration is present there exists a long run relationship between

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[18] See Tong (1983) for an early review on the application of TAR models to economic variables.

[19] β is known as the cointegrating vector.

them. Furthermore, by the Granger representation theorem we have that the short-run dynamics of the cointegrated variables may be represented by a Vector Error Correction Model –VECM.

If we consider the case of two I(1) cointegrated variables – $x_t = (x_{1,t}, x_{2,t})$–, then the corresponding VECM is given by

$$\Delta x_t = \alpha + \Pi x_{t-1} + \sum_{i=1}^{p-1} \Psi_i \Delta x_{t-i} + \varepsilon_t \tag{3.1}$$

where ε_t is an nx1 vector of serially uncorrelated error terms, α is an nx1 vector of intercept terms, and $\Pi = \gamma\beta$ and Ψ_i are nxn coefficient matrices where $rank(\Pi) = 1$. If we normalize the cointegrating vector so that $\beta_1 = 1$, then (3.1) is equivalent to

$$\begin{bmatrix} \Delta x_{1,t} \\ \Delta x_{2,t} \end{bmatrix} = \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix} + \begin{bmatrix} \gamma_1 \left(x_{1,t-1} - \beta_2 x_{2,t-1} \right) \\ \gamma_2 \left(x_{1,t-1} - \beta_2 x_{2,t-1} \right) \end{bmatrix} + \sum_{i=1}^{p-1} \Psi_i \Delta x_{t-i} + \varepsilon_t . \tag{3.2}$$

In this case γ_1 and γ_2 are interpreted as the speed of adjustment of any short-run disequilibrium for $x_{1,t}$ and $x_{2,t}$, respectively. Note that this speed of adjustment is constant regardless of the size and sign of the disequilibrium.

On the other hand, adjustment towards the long-run relationship does not need to be symmetric or the adjustment does not even need to exist for some periods, although a long-run relationship is present. Balke and Fomby (1997) proposed a nonlinear process for $z_t = \beta x_t$ which allow for non symmetric adjustments in the mean-reverting processes giving as a result richer short-run dynamics.

Balke and Fomby (1997) considered different three-regime Self-Exiting Threshold Autoregressive (STAR) processes for the cointegration error process. Among the proposed STAR processes, the most popular ones in the empirical literature are the Band-STAR and Equilibrium-STAR.

In the Equilibrium-STAR (EQ-TAR) specification the cointegrated error process tends towards equilibrium whenever outside a given region, i.e.,

$$\Delta z_t = \begin{cases} \phi^{(3)}(z_{t-1}) + \eta_t & \text{if } z_{t-d} > c^{(2)} \\ \eta_t & \text{if } c^{(1)} \leq z_{t-d} \leq c^{(2)}, \\ \phi^{(1)}(z_{t-1}) + \eta_t & \text{if } z_{t-d} < c^{(1)} \end{cases} \tag{3.3}$$

where d, $c^{(1)}$ and $c^{(2)}$ denote the delay parameter and the upper and lower threshold, respectively. Thus, whenever the d-periods delayed short-run disequilibrium is inside the region $\left[c^{(1)}, c^{(2)}\right]$, the cointegrating error follows a random walk. When the delayed short-run disequilibrium is outside the region $\left[c^{(1)}, c^{(2)}\right]$, then the process tends to get back to equilibrium[20]. If $c^{(1)} = c^{(2)} = c$, the (3.3) is called a symmetric EQ-TAR model.

In the Band-TAR model the cointegrated error process returns to an equilibrium band rather than to an equilibrium point when outside the region $\left[c^{(1)}, c^{(2)}\right]$, i.e.,

$$\Delta z_t = \begin{cases} \phi^{(3)}\left(z_{t-1} - \mu^{(3)}\right) + \eta_t & \text{if } z_{t-d} > c^{(2)} \\ \eta_t & \text{if } c^{(1)} \leq z_{t-d} \leq c^{(2)}, \\ \phi^{(1)}\left(z_{t-1} - \mu^{(1)}\right) + \eta_t & \text{if } z_{t-d} < c^{(1)} \end{cases} \tag{3.4}$$

where $\mu^{(1)}$ and $\mu^{(3)}$ are the lower and upper bounds of the band, respectively. For this model, if delayed short-run disequilibrium is in the region $\left[c^{(1)}, c^{(2)}\right]$, then the error follows a random walk. If delayed short-run disequilibrium is outside such region, the error tends to the boundaries of the band. If $c^{(1)} = c^{(2)} = c$, then we have a symmetric Band TAR process. And if $\mu^{(1)} = c^{(1)}$ and $\mu^{(2)} = c^{(2)}$, then (3.4) is said to be a continuous Band TAR model.

As in the linear case, if the elements of x_t are I(1) and there exist a vector of coefficients ($\beta = (\beta_1, \beta_2, \ldots, \beta_n)$) such that the linear combination $\beta x_t = z_t$ is stationary, then the elements of x_t are said to be cointegrated. Moreover, if there exists non-linear cointegration among the variables, there exists a nonlinear VECM representation of the system that corresponds to the short-run adjustment process to the long-run relationship. Such a representation will be referred as a Threshold Vector Error Correction Model (TVECM).

This chapter describes briefly the most popular econometric toolbox available to test for cointegration in the presence of a nonlinear process and a couple of techniques designed to detect cointegration in non-linear setups. The chapter also describes the estimation technique available to estimate TVECMs and specifications tests to choose among different TVECMs.

This chapter is divided into three sections. The first section discusses the cointegration tests available and their power against the null hypothesis of a three-regime threshold autoregressive behavior. A Monte Carlo study is presented to asses the power of the available cointegration tests. The second section briefly discusses different possible Threshold Vector Error Correction Models (TVECMs) available, and their estimation. The last part discusses specification tests available to help the researcher choose the TVECM that best describes a given data set.

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[20] For stability conditions of (3.3) see Balke and Fomby (1997).

3.1 Testing for Cointegration

Testing for cointegration implies two simultaneous tasks: finding the cointegrating vector and confirming that the resulting error process is stationary. In the case when the interest of cointegration is provoked by the possibility of a particular cointegration vector $\beta = b$, then testing for cointegration is reduced to test the stationarity of $z_t = bx_t$. We will focus on the case in which the cointegrating vector is known, and thus the issue of estimating the cointegrating vector will not be addressed.

Mainly, there have been two basic approaches to the problem of testing the null hypothesis of no cointegration. One approach is based on the time-series properties of the cointegrated residual; this approach implies a two-step procedure if the cointegrating vector is not known. The other approach is based on the rank of Π in (3.1); this approach typically implies only one step. In the next two sections the most popular cointegration tests available to test linear cointegration and non-linear cointegration are discussed.

Ideally, when nonlinear cointegration is considered, the goal is to test the null of no-cointegration against the alternative of threshold cointegration. Nevertheless, such a goal presents two methodological problems. First, a non-standard inference problem arises, since not only do unit roots appear under the null but also nuisance parameters are present in the alternative hypothesis that are not present under the null hypothesis, i.e., the thresholds. The second problem that arises is that the class of stationary threshold models that the cointegrating error term may present is too large to permit testing parametrically the no-cointegration null against a general threshold cointegration alternative. For instance the cointegrating error term may follow a two-regime or a three-regime TAR, or even a Band or EQ-TAR.

Thus the problem for testing for non-linear cointegration is not a conventional one. As a consequence of these problems two complementary approaches to the problem have arisen.

One approach, suggested by Balke and Fomby (1997), breaks the analysis into two parts: i) analyzing the global behavior of the series by testing the null of no-cointegration against the alternative of linear-cointegration with conventional tests and ii) studying whether the local behavior of the series is non-linear or not. The second approach, tests directly the null hypothesis of non-cointegration against the alternative of a particular parametric form of a non-linear stationary process, for example a two-regime TAR cointegrating error. In this section we review a set of linear cointegration tests that may be useful to test for cointegration in the presence of nonlinear cointegration and two cointegration tests that allow for nonlinear behavior under the alternative hypothesis.

3.1.1 Testing for Linear Cointegration

There is a wide variety of cointegration tests available to researchers, but perhaps the most commonly used are the Engle and Granger and the Johansen tests. In this section we will briefly describe the Engle and Granger, Johansen, and the Horvath and Watson cointegration tests.

The cointegration test suggested in the pioneering article by Engle and Granger (1987) is based on the OLS residuals of the regression of one of the series in x_t on the rest of series of x_t. Then, an Augmented Dickey-Fuller (ADF) test on the OLS residuals is suggested to test whether the resulting error term is stationary or not, and thus determine whether the series are cointegrated or not. The distribution of the ADF statistics on the residuals does not follow a conventional distribution, Engle and Yoo (1987) provide the relevant critical values.

The Engle and Granger Cointegration test is given by the following two steps:

Step 1: Regress $x_{i,t}$, for $i \in 1 \ldots n$ on, $x_{-i,t}$, where $x_{-it} = (x_{1,t}, x_{2,t}, \ldots, x_{i-1,t}, x_{i+1,t}, \ldots, x_{n,t})$ and denote the residuals sequence as $\hat{z}_t = x_{it} - \hat{\beta} x_{-i,t}$.

Step 2: Perform an ADF on the series $\widehat{z_t}$, i.e. estimate the following autoregression of $\widehat{z_t}$:

$$\Delta\widehat{z_t} = a_1\hat{z}_{t-1} + \sum_{i=1}^{p} a_{i+1}\Delta\hat{z}_{t-i} + \varepsilon_t ,$$

use a conventional t-statistic for $H_0 : a_1 = 1$, and compare it with Engle and Yoo's (1987) critical values.

If the interest in cointegration is motivated by the possibility of a particular known cointegrating vector, then we can use this cointegrating vector directly to construct the series $z_t = \beta x_{i,t}$ and use the standard ADF test to test for stationarity.

Another widely used cointegration test is the Johansen (1988) test. Johansen's multivariate approach is based on the fact that the rank of the matrix Π in (3.1) is equal to the number of cointegration vectors[21]. Denote the n characteristic roots of Π ordered from the smallest to the biggest by $\lambda_1, \lambda_2, \ldots, \lambda_n$. Thus, if the variables in x_t are not cointegrated, the rank of Π is zero and all the characteristic roots are zero.

Johansen suggests the following two statistics to test the number of characteristic roots that are significantly different from zero:

$$\lambda_{trace}(r) = -T\sum_{i=r+1}^{n} \ln\left(1-\hat{\lambda}_i\right) \tag{3.5}$$

and

$$\lambda_{\max}(r, r+1) = -T\ln\left(1-\hat{\lambda}_{r+1}\right), \tag{3.6}$$

where $\hat{\lambda}_i$ is the i-th ordered estimated value of the eigenvalue of the estimated matrix $\hat{\Pi}$. $\lambda_{trace}(r)$ tests the null hypothesis that the number of distinct cointegrating vectors is less than or equal to r. $\lambda_{\max}(r, r+1)$ tests the null of r cointegrating vectors against the alternative of r+1 cointegrating vectors.

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[21] For a proof of this statement see Johansen (1988).

The distributions of the λ_{trace} and $\lambda_{\max}$ statistics do not follow a conventional distribution; Johansen and Juselius (1990) provided critical values for five different nested cases: i) series in x_t have no deterministic trends and the cointegrating equations do not have intercepts, ii) series in x_t have no deterministic trends and the cointegrating equations have intercepts, iii) series in x_t have linear trends but the cointegrating equations have only intercepts, iv) both series in x_t and the cointegrating equations have linear trends, and v) series in x_t have quadratic trends and the cointegrating equations have linear trends.

The conclusions from the Johansen tests are very sensitive to the chosen lag length, thus it is important first to determine the appropriate lag length before testing for cointegration. The Johansen test involves the following steps:

Step 1: Using information multivariate criteria such as Akaike Information criterion (AIC) and/or Schwarz Bayesian information criterion (SBC), determine the optimal lag length of the VAR model for the undifferenced data.

Step 2: Estimate model (3.1) by Maximum Likelihood estimators and find the eigenvalues of $\hat{\Pi}$.

Step 3: Calculate $\lambda_{trace}(r)$ and $\lambda_{\max}(r,r+1)$ and compare with the appropriate critical values.

As in the two step procedure, when the cointegrating vector is known, we may incorporate that information into the test. Horvath and Watson (1995) proposed an alternative multivariate test for cointegration when the cointegrating vector is known. Their test is based on the idea that if there exists cointegration among the variables in x_t, then x_t has a VECM representation given in (3.1) and $\Pi = \gamma\beta$, where β are the cointegrating vectors. Thus, (3.1) can be rewritten as

$$\Delta x_t = \alpha - \gamma\,(\beta x_{t-1}) + \sum_{i=1}^{p-1} \Psi_i \Delta x_{t-i} + \varepsilon_t \,. \tag{3.7}$$

Under the null hypothesis of no-cointegration, the VECM representation does not exist, and thus $\gamma = 0$. Therefore, if the cointegrating vectors β are known, then the null hypothesis of no-cointegration is equivalent to the joint null hypothesis that $\gamma = 0$.

Thus, we may test the null hypothesis of no-cointegration with the standard seemingly unrelated regressions' Wald statistic given by

$$HW = \hat{\gamma}'\text{var}(\hat{\gamma})^{-1}\hat{\gamma} \tag{3.8}$$

where $\hat{\gamma}$ is the vector of equation-by-equation OLS estimators of γ, and $\text{var}(\hat{\gamma})$ is the OLS estimate of the covariance matrix of $\hat{\gamma}$. Horvath and Watson (1995) showed that the distribution of this Wald statistic, hereafter HW, does not follow a conventional distribution and provided tables with the appropriate critical values.

Thus the Horvath and Watson (1995) cointegration test implies the following steps:

Step 1: Estimate (3.7) by OLS equation by equation . And form a vector $\hat{\gamma}$ with the estimates of γ, and a matrix $\text{var}(\hat{\gamma})$ with the OLS estimate of the covariance matrix of $\hat{\gamma}$.

Step 2: Calculate the *HW* statistic and compare with the appropriate critical values.

3.1.2 Testing for Threshold Cointegration

There is not a non-linear cointegration test *per se* available yet; however there exist a few tests designed to test the null hypothesis of a unit root in an univariate autoregressive model against the alternative of a two-regime stationary TAR model, e.g., Gonzalez and Gonzalo (1997), Carner and Hansen (2001), Enders and Granger (1998), and Berben and van Dijk (1999). These tests are designed to have more power than the conventional unit root tests that do not consider the non-linear nature of the alternative hypothesis.

As in the linear cointegration test, if the cointegrating vector of interest is known ($\beta = b$), it seems natural to apply these nonlinear unit root tests to the constructed series $z_t = bx_t$. In this section we describe the Enders and Granger (1998) and Berben and van Dijk (1999) nonlinear unit root tests[22].

Enders and Granger (1998) considered the case when testing for a unit root in the symmetric continuous two-regime TAR model

$$\Delta z_t = \begin{cases} \phi^{(2)}(z_{t-1} - \gamma) + \eta_t, & \text{if } z_{t-1} > \gamma \\ \phi^{(1)}(z_{t-1} - \gamma) + \eta_t, & \text{if } z_{t-1} \le \gamma \end{cases} \tag{3.9}$$

where η_t is an i.i.d. random error. Under the null hypothesis of a unit root, $\phi^{(1)} = \phi^{(2)} = 0$. Enders and Granger suggested using the sample mean of z_{t-1} as an estimator for γ (the threshold) and testing the null of a unit root using the standard F-statistic from the OLS regression

$$\Delta z_t = \phi^{(1)}(z_{t-1} - \hat{\gamma}) I_{(z_{t-1} \le \hat{c})} + \phi^{(2)}(z_{t-1} - \hat{\gamma}) I_{(z_{t-1} > \hat{c})} + \sum_{j=1}^{k-1} \psi_j \Delta z_{t-j} + \eta_t \tag{3.10}$$

where $I_{(z_{t-1} \le \hat{\gamma})}$ takes the value of one if $z_{t-1} \le \hat{\gamma}$ and zero otherwise. Under the null hypothesis the distribution of the F-statistic follows a distribution that is a function of Brownian motion processes. Enders and Granger provided the appropriate critical values for the standard F-statistic.

Thus, the Enders and Granger test implies two steps:

Step 1: Estimate by OLS (3.10).

Step 2: Calculate the standard F-statistic test for testing the null $\phi^{(1)} = \phi^{(2)} = 0$ and compare with the appropriate critical values.

Enders and Granger (1998) employed a Monte Carlo experiment to compare the power of

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[22] The Gonzalez and Gonzalo (1997) and Carner and Hansen (2001) tests assume a stationary threshold variable (z_t) under the null of a unit root. Since, we are considering the cointegration residuals as the threshold variable

their test with the conventional ADF test when the true data generating process is generated by a symmetric continuous two-regime TAR model. The authors found that the ADF test had more power than their suggested statistic when the true data generating process is a two-regime TAR, even though their test takes in account the non-linear nature of the process.

Berben and van Dijk (1999) (BVD) suggest that the lower power of the Enders and Granger test may be due to the fact that the test uses a biased estimator of the threshold parameter under the alternative hypothesis. They argue that the Enders and Granger case corresponds to a particular case when the threshold happens to be exactly the mean, but this does not need to be the case.

BVD suggested a consistent estimator of the threshold – γ in (3.10) – and then the use of a conventional F-statistic to test the null hypothesis that $\phi^{(1)}=\phi^{(2)}=0$. In addition, the authors found that the distribution of the F-statistic has a mix of "Dickey-Fulller" and "sup-Wald" characteristics, so that critical values are not conventional. However, BVD provided the appropriate critical values.

The BVD test implies the following steps[23]:

Step 1: (Finding an unbiased estimate for the threshold) For given $\tau_0, \tau_1 \in (0,1)$, find

$$\hat{\tau}=\underset{\tau \in T}{\operatorname{argmin}}\left\{\sigma_n^2(\gamma(\tau))\right\}, \tag{3.11}$$

where $\sigma_n^2(\gamma(\tau))$ is the variance of the OLS residual of the regression given by (3.10) setting $\hat{\gamma} \equiv \hat{\gamma}(\tau)=(1-\tau)z_{(1)}+\tau z_{(n)}$, and $z_{(k)}$ is the k-th order statistic of z_t.

Step 2: Calculate the standard F-statistic test for testing the null $\phi^{(1)}=\phi^{(2)}=0$ and compare with the appropriate critical values.

Berben and van Dijk (1999) used a Monte Carlo experiment to show that the BVD test

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these two tests are not applicable.

[23] For moe details on the BVD test we remit the reader to the original paper.

has much more power than the Enders and Granger and the ADF test when the true data generating process is a two-regime TAR.

Note that the cointegration tests[24] discussed before are designed to test the null of no-cointegration against the alternative of a stationary error that follows a two-regime TAR process. As pointed out by Lo and Zivot (2001), it is expected that these tests have some power against the alternative of a three regime TAR process for cointegrating error, since Bai (1997) showed that if a three-regime TAR is the true model, the least-squares estimate of the threshold on the misspecified two-regime model will be consistent for one of the thresholds. Therefore, one of the estimated autoregressive coefficients in (3.10) should be less than zero and this will give the nonlinear tests more power than the linear tests.

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[24] Strictly speaking the tests discussed in this subsection are unit root tests. But as argued before, these tests may be used to test for cointegration if the cointegrating vector of interest is known.

3.2 Monte Carlo Study

Balke and Fomby (1997) used a Monte Carlo study to show that conventional cointegration tests, such as the Engle and Granger cointegration test, have moderate power when the delay parameter is known to be equal to one and the true data generating process of the cointegrating errors is a continuous and symmetric three regime TAR. Lo and Zivot (2001), using the same experimental design as Balke and Fomby[25] (1997), compared the power of the ADF test (with a known cointegrating vector), the HW test, the Enders and Granger test, and the BDV test. The authors found that the tests for no-cointegration for the EQ specification have more power than those for the Band specification. Furthermore, the authors found that the power is higher for smaller values of the thresholds.

Lo and Zivot (2001) also found that among the linear-cointegration tests, the HW test possesses about twice as much power as the ADF test. However, the Enders and Granger and BVD tests possess more power than the HW test. Finally, they found that the BVD test has the best power among all the tests considered for moderate sample sizes[26].

Falk and Xu (2002) used a Monte Carlo experiment to compare the performance of the Engle and Granger, the Enders and Siklos (2000), and the Johansen tests when the true data generating process of the cointegrating error is a two-regime TAR and the cointegrating vector is unknown. The authors found that the power of the Johansen test is lower than the other two tests, and that the Engle and Granger test has more power than the Enders and Siklos (2000) test for significance levels of 10% and 5%.

In this subsection we present a Monte Carlos experiment to examine the power of the cointegration tests described above for significance levels of 5% when the cointegration errors are generated from a three regime Band or EQ-TAR with symmetric and asymmetric

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[25] They set $\phi^{(1)} = \phi^{(3)} = -.6$ in (3.3) and (3.4).

[26] Sample sizes of 250 or 500.

thresholds and an unknown delay parameter.

Unlike all the Monte Carlo experiments reported in the literature this study considers the case of a misspecified delay parameter. Note that the non-linear tests are designed for a delay parameter of one. Thus it is not clear what the effect of a misspecified delay parameter on the power of cointegration tests is. Another characteristic shared by previous Monte Carlo studies is that the considered thresholds are always symmetric; we will allow asymmetric thresholds in our study. This subsection is divided into two parts; the first one describes the design of the experiment. The second part presents the results.

3.2.1 Design of the Experiment

In this subsection a Monte Carlo Study is presented. The study is designed to investigate the effects of a misspecified delay parameter (d), different AR coefficients in the outer regimes of the cointegration error ($\phi^{(1)}$ and $\phi^{(2)}$), and asymmetric thresholds on the rejection rate of the previously discussed cointegration tests.

We consider the following cointegration system:

$$x_{1,t} = \beta x_{2,t} + z_t \tag{3.12}$$

where $x_{1,t}$ and $x_{2,t}$ are I(1) without a drift, $\beta = 1$[27], and z_t follows (3.3) in the case of an EQ-TAR cointegrated error or (3.4) in the case of a Band-TAR cointegrated error. In both cases η_t is generated as i.i.d. $N(0,1)$.

First, following Balke and Fomby (1997) and Lo and Zivot (2001), the Monte Carlo study is carried out using symmetric thresholds $c^{(1)} = c^{(2)} = 3,5,10$. In a second stage, the

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[27] Balke and Fomby (1997) and Lo and Zivot (2001) used in their Monte Carlo studies a cointegrating vector $(1,-2)$

study is done using the following asymmetric thresholds $\left(c^{(1)},c^{(2)}\right)=(-5,3)$, $(-3,5)$, $(-10,5)$, and $(-3,10)$.

For all the cases, 1,000 samples of sizes $T=100$, 250, and 500 and outer regimes AR coefficients $\phi^{(1)}=\phi^{(2)}=$ -0.1, -0.6, and -0.9 are generated from both the Band and EQ versions of (3.12)[28] and the discussed cointegration tests are performed. For the ADF, Engel and Granger, Enders and Granger, and BVD tests the lag length is selected by minimizing the AIC. In the case of the Johansen and HW multivariate tests, a VAR is first estimated to select the appropriate lag length by minimizing the multivariate version of the AIC – (3.27) –. The rejection rates of each test are recorded and reported in Appendix 1.

3.2.2 Results of the Monte Carlo Study

The results for the designed Monte Carlo Study are reported in Appendix 1. Tables A1 - A4 present the rejection frequency of the cointegration tests for the different cases considered. Tables A5 – A7 present the two tests with the largest power for each one of the considered cases.

From the results of the Monte Carlo study we can conclude, as did Balke and Fomby (1997) and Lo and Zivot (2001), that the rejection frequencies (the power) for all the tests and cases are higher for the EQ specification than for the Band specification.

In general, the power of the cointegration tests is larger, the closer the absolute value of the outer regime coefficients are to one ($\left|\phi^{(1)}\right|=\left|\phi^{(2)}\right|=|\phi|$), and thus the process displays less persistent behavior. Thus we may conclude that the more periods needed by the process to return to a state inside the thresholds, the less power the cointegration tests will possess. It is important to point out the relatively low powers – 5% to 30% – that pertain to all the tests for symmetric and asymmetric models when the outer regime AR coefficients are equal to -0.1 ($\phi = -.1$).

It is important to mention that for the relatively low-power cases, the null of no-cointegration is not rejected in a large proportion of the cases – approximately 80% – by all the cointegration tests except for one. The tests that do reject the null hypothesis vary always, so that no systematic pattern could be found.

Another interesting finding from the study is that the effect of a misspecified delay parameter (*d*) varies according to the statistic. For example, the power of the ADF statistic decreases as the true *d* increases for both the case of a symmetric and asymmetric Band specification and symmetric and asymmetric EQ specification. Another statistic that presents a decrease in its power as *d* increases is the HW. On the other hand, the power of the Engel and Granger test is fairly stable, no matter the delay parameter or the specification, the power of this test is consistently between 50% and 60%. The BVD and the Johansen statistics also present a fairly stable behavior with respect to changes in the delay parameter.

For symmetric models the power of all the tests decrease, *ceteris paribus*, as the distance between the thresholds gets larger; this same result was found by Balke and Fomby (1997) and Lo and Zivot (2001). This result is also true for the asymmetric models. An interesting result for asymmetric models is that, *ceteris paribus*, the power is similar for same distance of the thresholds, e.g. when the thresholds are (-5,3) or (-3, 5) the power of all the tests are very similar.

Unlike previous Monte Carlo studies, this study does not suggest that one test or a set of tests performs better than the others in all situations. Tables A5 – A10 present the tests with the largest power for each of the cases. This study suggests that there is not a single test that outperforms the others when testing cointegration in the presence of a three-regime VAR

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[28] With cointegrating vector [1,-1].

cointegrating error. For example, for small samples – T=100 – the Engel and Granger test performs better than the others when we consider models with $\phi = -.1$. However, if models with $\phi = -.6$ are considered, then the tests presenting the largest powers are the HW test for the Band specification and the ADF for the EQ specification.

The only test that is always outperformed by the others or does as well as the others is the Enders and Granger test. The reason for this result may be the fact, pointed out by Berben and van Dijk (1999), that the Enders and Granger test uses a biased estimator of the threshold parameter under the alternative hypothesis. Recall that the argument to use tests that utilize two-regime TAR is that according to Bai (1997) the least-squares estimate of the threshold on the misspecified two-regime model will be consistent for one of the thresholds. Since the Enders and Granger test uses a biased estimator of the thresholds, this may result in the poor performance of the test.

To conclude this section we suggest that, in practice, when facing the task of testing cointegration in the potential presence of three-regime threshold cointegration, one should use a set of cointegration tests and base the final decision in favor or against cointegration on the consensus of the different tests. Clearly more theoretical work has to be done to develop a cointegration test that permits a three-regime TAR cointegrating error, since the tests currently available do not perform very well in all possible situations.

3.3 Estimating a Threshold Vector Error Correction Model

In practice, the researcher faces different problems when considering non-linear data generating processes. First, it is necessary to determine whether a non-linear model better fits the data set than a simpler linear model. Once evidence in favor of non-linearity is found, it is necessary to determine what kind of non-linear model better fits to the data, for example a two-regime or a three-regime model. And after the number of regimes has been determined, then different competing models are available to the researcher. In this subsection we first describe the two and three regime non-linear models and the procedures to estimate the models and determine which model better fits the data. The first part discusses the generalities of a non-linear TVECM. The second part discusses the way of estimating the different non-linear models. Finally, the third part describes the specifications tests currently available

3.3.1 The Threshold Vector Error Correction Model, The Band-Threshold Vector Error Correction Model, and The Equilibrium-Threshold Vector Error Correction Model

If cointegration between the series in x_t is determined and thus a long-run relationship between the series in x_t exists; then, by Granger's Representation Theorem, there exists a corresponding Vector Error Correction Model (VECM) that represents the short-run adjustment of the system to short run disequilibria. In the case of a nonlinear behavior of a stationary non-linear cointegrating error, then the corresponding VECM is known as a Threshold Vector Error Correction Model (TVECM).

A threshold VAR (TVAR) model for x_t with *p* lags, *k* regimes, threshold variable z_t and delay parameter *d* is given by

$$x_t = \alpha_0^{(j)} + \sum_{i=1}^{p} A_i^{(j)} x_{t-i} + \varepsilon_t^{(j)}, \qquad \text{if } c^{(j-1)} \le z_{t-d} \le c^{(j)}, \qquad (3.13)$$

for j=1,2,...,k., where $A_0^{(j)}$ is a constant column vector of size 2, $A_i^{(j)}$ is a 2x2 matrix of constants for i=1,...p, $-\infty < c^{(0)} < c^{(1)} < \ldots < c^{(k)} < \infty$ and $\varepsilon_t^{(j)}$ is a serially uncorrelated error term. For simplicity, let's consider the case where $x_t = (x_{1,t}, x_{2,t})$ and let each series in x_t be an I(1) series. In addition, let the threshold variable z_t be known and stationary; however the delay parameter d, lag length p, and threshold values $c^{(0)}$, $c^{(1)}$,..., $c^{(k)}$ may be unknown.

If x_t is a cointegrated random vector, then (3.13) can be reparametarized as

$$\Delta x_t = \alpha_0^{(j)} + \Pi^{(j)} x_{t-1} + \sum_{i=1}^{p-1} \Psi_i^{(j)} \Delta x_{t-i} + \varepsilon_t^{(j)}, \qquad \text{if } c^{(j-1)} \le z_{t-d} \le c^{(j)}, \qquad (3.14)$$

where $\Pi^{(j)} = \sum_{i=1}^{p} A_i^{(j)} - I_2$, and $\psi_i^{(j)} = -\sum_{l=i+1}^{p} A_l^{(j)}$. Furthermore, if we assume the cointegrating vector is known so that $\beta^T = (1, -b)$, then

$$\Pi^{(j)} = \gamma^{(j)} \beta^T = \begin{bmatrix} \gamma_1^{(j)} \\ \gamma_2^{(j)} \end{bmatrix} (1, -b) \qquad (3.15)$$

and the TVECM representation of the system is given by

$$\Delta x_t = \alpha_0^{(j)} + \begin{bmatrix} \gamma_1^{(j)} \\ \gamma_2^{(j)} \end{bmatrix} (1, -b) x_{t-1} + \sum_{i=1}^{p-1} \psi_i^{(j)} \Delta x_{t-i} + \varepsilon_t^{(j)}, \qquad \text{if } c^{(j-1)} \le z_{t-d} \le c^{(j)}. \quad (3.16)$$

Note that the adjustment factor toward the long-run equilibrium relationship ($\gamma_i^{(j)}$) is regime specific.

(3.16) can be written as

$$\Delta x_t = \gamma^{(j)} \left[\beta x_{t-1} - \mu^{(j)} \right] + \sum_{i=1}^{p-1} \psi_i^{(j)} \Delta x_{t-i} + \varepsilon_t^{(j)}, \qquad \text{if } c^{(j-1)} \le z_{t-d} \le c^{(j)} \qquad (3.17)$$

for j=1,2,...,k.; where $\mu^{(j)}$ is the regime specific mean of the cointegrating relation $\beta^T x_t$ and $\alpha_0^{(j)} = -\gamma^{(j)} \mu^{(j)}$.

Now, consider the special case of the TVECM in (3.17) where: i) k=3 (a three-regime TVAR) and ii) the threshold variable is set to be equal to the residual from the cointegrating relationship $\beta^T x_t$, i.e., $z_{t-1} = \beta^T x_{t-1}$. Thus, (3.17) becomes

$$\Delta x_t = \begin{cases} \gamma^{(3)}\left(z_{t-1} - \mu^{(3)}\right) + \sum_{i=1}^{p-1} \psi_i^{(3)} \Delta x_{t-i} + \varepsilon_t, & \text{if} \quad z_{t-d} > c^{(2)} \\ \gamma^{(2)}\left(z_{t-1} - \mu^{(2)}\right) + \sum_{i=1}^{p-1} \psi_i^{(2)} \Delta x_{t-i} + \varepsilon_t, & \text{if } c^{(1)} \le z_{t-d} \le c^{(2)} \\ \gamma^{(1)}\left(z_{t-1} - \mu^{(1)}\right) + \sum_{i=1}^{p-1} \psi_i^{(1)} \Delta x_{t-i} + \varepsilon_t, & \text{if} \quad z_{t-d} < c^{(1)} \end{cases}. \tag{3.18}$$

(3.18) is known as a three-regime TVECM, which implies the following three-regime TAR process for the cointegrated error – z_t –:

$$\Delta z_t = \begin{cases} \phi^{(3)}\left(z_{t-1} - \mu^{(3)}\right) + \eta_t, & \text{if} \quad z_{t-d} > c^{(2)} \\ \phi^{(2)}\left(z_{t-1} - \mu^{(2)}\right) + \eta_t, & \text{if } c^{(1)} \le z_{t-d} \le c^{(2)} \\ \phi^{(1)}\left(z_{t-1} - \mu^{(1)}\right) + \eta_t, & \text{if} \quad z_{t-d} < c^{(1)} \end{cases} \tag{3.19}$$

where η_t is a random error term.

Two special cases of the TVECM are the Band-TVECM and the Equilibrium-TVECM (EQ-TVECM). For the Band-TVECM $\gamma^{(2)} = 0$ in (3.18) so that a Band-TVECM is given by

$$\Delta x_t = \begin{cases} \gamma^{(3)}\left(z_{t-1} - \mu^{(3)}\right) + \sum_{i=1}^{p-1} \psi_i^{(3)} \Delta x_{t-i} + \varepsilon_t, & \text{if} \quad z_{t-d} > c^{(2)} \\ \varepsilon_t, & \text{if } c^{(1)} \le z_{t-d} \le c^{(2)} \\ \gamma^{(1)}\left(z_{t-1} - \mu^{(1)}\right) + \sum_{i=1}^{p-1} \psi_i^{(1)} \Delta x_{t-i} + \varepsilon_t, & \text{if} \quad z_{t-d} < c^{(1)} \end{cases}. \tag{3.20}$$

In the middle regime the error is not cointegrated but is I(1). As stated by Lo and Zivot (2001), a sufficient condition for the stability of (3.20) is that the outer regimes are stable, i.e., $\left|1 + \gamma_1^{(j)} + \gamma_2^{(j)}\right| < 1$ for j=1,3. Note that (3.20) implies a cointegrated error that follows a Band-TAR as in (3.4).

Thus, a Band-TVECM implies that whenever the d-period delayed short-run disequilibrium is inside the region $\left[c^{(1)}, c^{(2)}\right]$, then the cointegrated error follows a random walk. When the delayed short-run disequilibrium is outside the region $\left[c^{(1)}, c^{(2)}\right]$, then the process tends to get back to the boundaries of the band given by $\left[\mu^{(1)}, \mu^{(2)}\right]$.

If $c^{(1)} = -c^{(2)} = c$, then (3.20) is said to be symmetric Band-TVECM. If $c^{(1)} = \mu^{(1)}$ and $c^{(2)} = \mu^{(2)}$, then (3.20) is known as a continuous Band-TVECM. If $\mu^{(1)} = \mu^{(3)} = 0$ for the Band-TVECM – (3.20) –, then (3.18) becomes an EQ-TVECM. Thus, the Band-TVECM is nested in the TVECM and the Equilibrium-TVECM is nested in the Band-TVECM.

3.3.2 Estimating a TVECM

In this section we describe the techniques of sequential conditional Multivariate Least Squares[29] to estimate two and three-regime TVECM. Let's consider the estimation of an unrestricted two-regime TVECM given by

$$\Delta x_t = \begin{cases} \pi^{(2)} + \gamma^{(2)} z_{t-1} + \sum_{i=1}^{p-1} \psi_i^{(2)} \Delta x_{t-i} + \varepsilon_t & \text{if} \quad z_{t-d} \geq c \\ \pi^{(1)} + \gamma^{(1)} z_{t-1} + \sum_{i=1}^{p-1} \psi_i^{(1)} \Delta x_{t-i} + \varepsilon_t & \text{if} \quad z_{t-d} < c \end{cases} . \tag{3.21}$$

(3.21) can be rewritten in a more compact way as

$$\Delta x_t = y_{t-1}\Theta^{(1)} I_t(c,d) + y_{t-1}\Theta^{(2)}\left[1 - I_t(c,d)\right] + \varepsilon_t \tag{3.22}$$

where $y_{t-1} = \left(1, z_{t-1}, \Delta x_{t-1}, \ldots, \Delta x_{t-p+1}\right)$, $z_{t-1} = \beta' x_{t-1}$ with β known, $\Theta^{(j)}$ is a $2(k+1)$x2 matrix

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[29] Tong (1983) suggested this approach, Hansen (1999) retook this approach to the problem of estimating a multi-equation system. Chan and Tsay (1998) and Berben and van Dijk (1999) discussed the estimation of univariate TAR models.

of coefficients, and $I_t(c,d) = I_{(z_{t-d}<c)}$. As mentioned before $I_{(\bullet)}$ represents the indicator function such that $I_{(A)} = 1$ if A is true and $I_{(A)} = 0$ otherwise.

Without lost of generality, suppose the delay parameter to be bounded by an integer $\bar{d}$. Without lost of generality, it is also possible to restrict the potential values of c to the observed values of z_{t-d}, since the threshold value c only appears in (3.22) through the indicator function $I_{(z_{t-d}<c)}$. Hansen (1999), based on asymptotic theory, suggested that c be restricted so that the number of observations included in each regime (T_i) satisfies $\lim_{T\to\infty}(T_i/T) \geq \tau$, for some $\tau \in (0,1)$. Specifically Hansen (1999) suggested setting $\tau = 0.1$.

As mentioned before, (3.21) can be estimated by Sequential Conditional Multivariate Least Squares (SCMLS). SCMLS implies first the estimation of $\Theta^{(1)}$ and $\Theta^{(2)}$ by multivariate least squares, conditional on a given pair (c,d), so that the following residual sum of squares can be found

$$S_2(c,d) = trace\left[\hat{\Sigma}_2(c,d)\right], \tag{3.23}$$

where $\hat{\Sigma}_2(c,d)$ represents the multivariate least squares estimate of $\text{var}[\varepsilon_t]$ conditional on (c,d). Then the estimates of c and d ($\hat{c}$ and $\hat{d}$) are found by minimizing $S_2(c,d)$ with respect to all possible values of c and d[30]. Thus the estimate for $\Theta^{(1)}$ and $\Theta^{(2)}$ are $\hat{\Theta}^{(1)}(\hat{c},\hat{d})$ and $\hat{\Theta}^{(2)}(\hat{c},\hat{d})$.

Thus the steps to estimate (3.22) by SCMLS are:

Step 1: For all possible pairs[31] (c,d) find the Multivariate Least Squares estimates for $\Theta^{(1)}$

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[30] The possible values of d are restricted to $(1,\bar{d})$ while the values for c are restricted to the percentiles of z_t such that $\lim_{T\to\infty}(T_i/T) \geq \tau$.

[31] See previous footnote.

and $\Theta^{(2)}$. For each pair (c,d), calculate, $S_2(c,d) = trace\left[\hat{\Sigma}_2(c,d)\right]$.

Step 2: Find

$$\left(\hat{c},\hat{d}\right) = \underset{(c,d)}{\operatorname{argmin}} S_2(c,d) \tag{3.24}$$

Step 3: Find $\hat{\Theta}^{(1)}\left(\hat{c},\hat{d}\right)$ and $\hat{\Theta}^{(2)}\left(\hat{c},\hat{d}\right)$.

Tsay (1998) showed that under mild regularity conditions: i) the SCMLS estimates ($\hat{\Theta}^{(1)}\left(\hat{c},\hat{d}\right)$, $\hat{\Theta}^{(2)}\left(\hat{c},\hat{d}\right)$, $\hat{c}$, and $\hat{d}$) are strongly consistent, ii) the estimates of $\Theta^{(1)}$ and $\Theta^{(2)}$ are asymptotically normally distributed independent of *c* and *d*, and iii) $\hat{c}$ and $\hat{d}$ converge at rate *T*.

Now consider the estimation of an unrestricted three-regime TVECM such as (3.18) or equivalently

$$\Delta x_t = y_{t-1}\Theta^{(1)}I_t^{(1)}(c,d) + y_{t-1}\Theta^{(2)}I_t^{(2)}(c,d) + y_{t-1}\Theta^{(3)}I_t^{(3)}(c,d) + \varepsilon_t \tag{3.25}$$

where $c = \left(c^{(1)}, c^{(2)}\right)$ and $I_t^{(j)}(c,d) = I_{\left(c^{(j-1)} < z_{t-d} < c^{(j)}\right)}$. Model (3.25) can be estimated using a procedure similar to the three-step procedure just described above. First, conditional on (c,d), estimate $\Theta^{(1)}$, $\Theta^{(2)}$, and $\Theta^{(3)}$ by multivariate least squares giving the residual sum of squares $S_3(c,d)$. Then the estimates for (c,d) are found by minimizing $S_3(c,d)$ by a three-dimensional grid search.

As mentioned before, the EQ-TVECM and the Band-TVECM are restricted versions of the unrestricted three-regime TVECM – (3.25). The above procedure needs to be modified to take into account the restricted nature of the EQ- and Band-TVECM, since due to the cross-equation restrictions multivariate least squares is no longer efficient, although it is consistent. Lo and Zivot (2001) recommend a modified procedure, so that the cross-equation restrictions are considered.

Lo and Zivot (2001) suggested first to estimate (c,d) from the unrestricted model (3.25), since the estimates for (c,d) are super consistent in the unrestricted model. Then, using the estimated (c,d), the corresponding restricted three regime model can be estimated using Zellner's seemingly unrelated regression (SUR) approach, i.e.,

$$\left[\Theta_{SUR}^{(1)}\left(\hat{c},\hat{d}\right),\Theta_{SUR}^{(2)}\left(\hat{c},\hat{d}\right),\Theta_{SUR}^{(3)}\left(\hat{c},\hat{d}\right)\right]=\underset{\Theta^{(1)},\Theta^{(2)},\Theta^{(3)}}{\operatorname{argmin}}\left\{\frac{1}{2}\log\left(\left|\Sigma_{sur3}\left(\hat{c},\hat{d}\right)\right|\right)\right\} \tag{3.26}$$

where $\Sigma_{SUR3}\left(\hat{c},\hat{d}\right)=\frac{1}{T}\sum_{i=1}^{T}\varepsilon_t\left(\hat{c},\hat{d}\right)\varepsilon_t\left(\hat{c},\hat{d}\right)'$.

Finally, to select the lag length of any of the TVECM discussed above, a multivariate version of the Akaike Information Criterion (AIC) and Schwartz Bayesian Criterion (SBC) may be used. In other words, the lag length may be selected by minimizing the AIC or/and the SBC. The multivariate versions of the AIC and SBC are given in the following formulas:

$$AIC=T\log|\Sigma|+2N \tag{3.27}$$

$$SBC=T\log|\Sigma|+N\log(T) \tag{3.28}$$

where $|\Sigma|$ and N represent the determinant of the covariance matrix of the residuals and the total number of parameters estimated in all equations.

3.3.3 Specification tests

As mentioned in section 2.1, there is not a threshold cointegration test *per se*. From the cointegration tests it is not possible to determine if a nonlinear VECM or a linear VECM better represents the short run behavior of cointegrated variables. Furthermore, in case a nonlinear short-run behavior is indeed present, the cointegration tests do not give any evidence in favor of a two, three or higher order regime TVECM. In this section we, summarize the specifications tests to identify a TVECM that better fits a given data set.

Once cointegration between the series in x_t has been determined, it is possible to proceed to determine what type of VECM better fits the data —linear, two-regime TVECM, unrestricted three-regime TVECM, Band-TVECM or EQ-TVECM. Lo and Zivot (2001) suggested a Likelihood Ratio model-specification test for nested models based on Hansen (1999). The suggested test for nested models to test the null hypothesis of a restricted model (R) versus the alternative hypothesis of an unrestricted model (U) is given by:

$$LR_{R.U} = T\left(\ln\left(\left|\hat{\Sigma}_U\left(\hat{c},\hat{d}\right)\right|\right) - \ln\left(\left|\hat{\Sigma}_R\left(\hat{c},\hat{d}\right)\right|\right)\right) \tag{3.29}$$

where $\hat{\Sigma}_R\left(\hat{c},\hat{d}\right)$ and $\hat{\Sigma}_U\left(\hat{c},\hat{d}\right)$ denote the estimated residual covariance matrices from the restricted and unrestricted models, respectively.

Thus the $LR_{U.R}$ statistic may be use to test several null hypothesis. The $LR_{R.U}$ may be used as a tests for linearity. For instance, $LR_{1.2}$ may be used to test the null hypothesis of a linear (one-regime) VECM (1) versus the alternative of a two-regime TVECM (2). Another linearity test is given by $LR_{1.3}$, where the restricted model is the linear model (1) and the unrestricted model is the three-regime TVECM (3). In addition, the $LR_{R.U}$ statistic may be used to determine if an unrestricted TVECM or a Band-TVECM (4) or an EQ-TVECM (5) better fit the data. $LR_{5.4}$ may be used to test the null hypothesis of an EQ-TVECM (5) versus the alternative hypothesis of a Band-TVECM (4).

Unfortunately, the distribution of $LR_{U.R}$ does not follow a conventional χ^2 distribution; hence, Hansen (1999) suggested the use of bootstrap methods to calculate p-values for the statistic. Hansen's bootstrap method applied to our problem implies the following steps to find the p-value for a given statistic, in this case the LR:

<u>Step 1:</u> Generate a random sample e_t^* for $t = 1,\ldots T$ by sampling with replacement from the estimated residuals from the unrestricted model.

<u>Step 2:</u> From the restricted estimated model, generate recursively a sample x_t^* for $t = 1,\ldots T$,

using as fixed initial conditions $(x_0, x_{-1}, \ldots, x_{-h+1})$, where $h = \max\{p, d\}$.

Step 3: Calculate the statistic from the generated sample x_t^* using the same formula as used to calculate the statistic on the observed series.

Step 4: Repeat the process many times[32].

Step 5: Calculate the percentage of simulated statistics which exceed the observed statistic. This is the bootstrap p-value.

Another specification test is suggested by Hansen (1999) is a sup-F-type test –also known as Super Wald test– given by

$$F_{R,U} = T\left(\frac{S_R - S_U}{S_u}\right) \tag{3.30}$$

where S_R and S_U represent the sum of squared residuals from the restricted and unrestricted models, respectively. As in the case of the likelihood ratio specification tests, the distribution of this sup-F statistics does not follow a conventional distribution due to the presence of nuisance parameters under the alternative hypothesis. To find p-values for the observed statistic Hansen suggested the use of simulation techniques such as the bootstrap method described above.

To conclude, it is important to emphasize that the $LR_{R,U}$ and $F_{R,U}$ tests may be used as a linearity test, where the restricted model is a linear VECM (1) and the unrestricted model may be a two-regime TVECM (2) or a three-regime TVECM (3). Similarly $LR_{R,U}$ and $F_{R,U}$ may be used as specification tests. In Chapter 4, the estimation and specification tests techniques described in this chapter will be applied to the problems of sustainability of a fiscal policy and a current account for quarterly data of the G-7 countries.

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[32] For this research, we will repeat the process 2000 times.

4 Empirical Analysis: Sustainability of the Fiscal Policies and Current Accounts of the G7 Countries

The discussion in Chapter 2 showed that there is mixed evidence about the sustainability of the fiscal policies in different countries and, to a less significant degree, on the sustainability of the current accounts of those countries. In principle one may expect that the current account or fiscal deficit may not be sustainable for a couple of periods, since once the economic agents involved realize the magnitude of the accumulated debt they will make the required adjustment to place the economy back on a sustainable path. Thus, the use of different sample sizes will capture the presence of a sustainable or non-sustainable scenario depending on the size and start of the sample, but the econometric tools previously available to economists did not allow them to precisely capture this idea.

For example, if a fiscal policy becomes unsustainable we may expect the fiscal authority to make adjustments in the structure of tax revenues and/or expenditures in order to return the fiscal deficit to a sustainable path. These adjustments imply a costly process that cannot be done instantly and require political bargaining in different levels of the government. Thus, we do not expect an instant adjustment process to revert the system to the long-run relationship. However, once the deviation achieves a "threshold", the situation is unmanageable and the authority will incur the cost needed to reach an adjustment.

It is instructive to think as Hamilton and Flavin (1986) suggested: that "One might want to admit the possibility of a change in regime in which the government budget had been expected to be balanced in present-value terms up until some date t and only after that date was a permanent deficit introduced" (p. 816). And, after a period $t+k$ an adjustment to a sustainable situation is introduced. This idea may be suggested by observing the time series of tax revenues and current expenditures for the U.S. depicted in Graph 1.

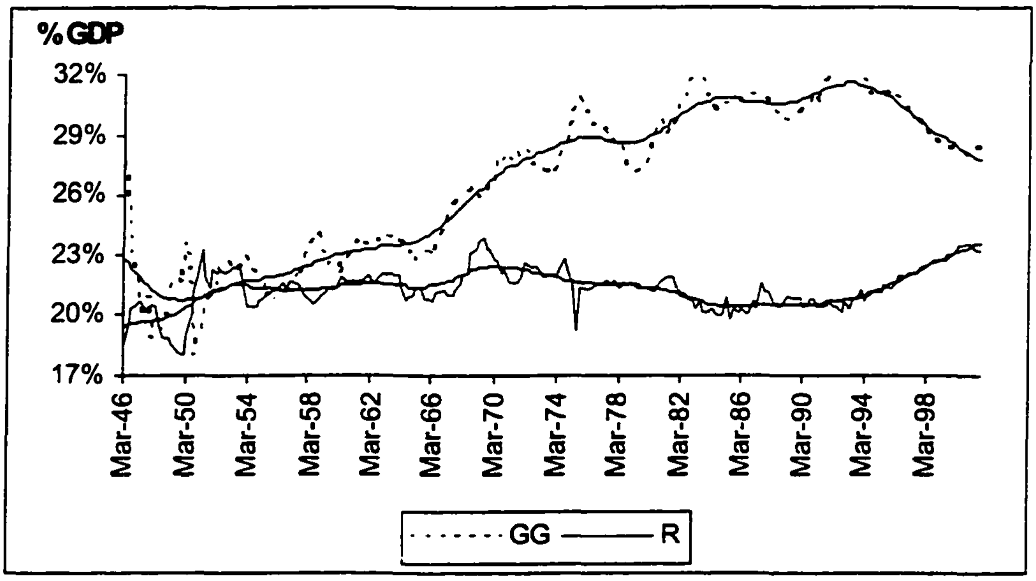

Figure 4.1. Revenues and Expenditures of the US Federal Government (Quarterly data as % of GDP)

According to Hakkio and Rush (1991) a sustainable fiscal policy implies that total revenues and expenditures (including the service of the debt) have to be cointegrated with a cointegrating vector [1,-1]. Similarly, Husted (1992) showed that a sustainable current account implies that exports and imports minus the service to the foreign debt have to be cointegrated, with a cointegrating vector [1,-1]. Traditional cointegration methods (linear cointegration) imply that there exists a linear error process representing the adjustment of the system to any short run disequilibria. The adjustment process is therefore linear and symmetric around a mean with a constant speed of adjustment. On the other hand, the notion of threshold cointegration captures perfectly this idea of a long-run relationship between economic variables turning "on" and "off" in the short-run.

This chapter presents the empirical results of the sustainability analysis of the fiscal policies and the current accounts of the G-7 countries (Canada, France, Germany, Japan, The United Kingdom, The United States, and Italy) using threshold cointegration techniques that may capture the possibility of a long-run relationship appearing only after a threshold is reached. The theoretical framework and the econometric tools used in this chapter were discussed in Chapter 2 and Chapter 3, respectively.

Throughout the analysis quarterly data will be used. The data set is from the International Financial Statistics data base published by the International Monetary Fund (IMF). This database has the advantage that it uses the same methodology to measure the economic variables across countries. For the case of fiscal data, the central governments report their revenues (Tax revenues without Social Security contributions plus other revenues) and expenditures (debt service plus other expenditures) following the IMF standards. For the case of the trade data; the relevant country specific authority provides the data to the IMF. In all cases, the IMF consolidates the data and reports, with a standardized methodology, the variables that we will use in our study: gross domestic product – GDP–, central government revenues and expenditures, total exports and imports of goods and services, and the service of the foreign private and public debt.

One of the disadvantages of using the IMF data set is that only seasonal adjusted data[33] are reported. Additionally, for the fiscal data the available samples are not of the same size for all 7 countries; each country began to report its quarterly information on different dates and some countries like Japan stopped reporting this quarterly data one decade ago. For our sustainability analysis we will consider the relevant economic variables as a share of the GDP in order to account for the economy's growth. For the analysis, we have divided the German data in two periods: before unification and after. We have done so to avoid a considerable jump presented in the economic variables once the reunification process began.

This Chapter is divided into two parts. The first part considers the sustainability of the fiscal policies of the G-7 countries, while the second section analyzes the sustainability of the current accounts.

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[33] The IMF seasonal adjusts the series using X11 methodology.

4.1 Sustainability of the Fiscal Policy of the G-7 Countries

In this section the sustainability of the fiscal policies of the G-7 countries is studied. The quarterly fiscal data available from the IMF data do not cover the same period for the countries under study. The period of studies are 1976:1–1995:2, 1970:1–2001:2, 1963:1–1991:2, 1991:1–2001:1, 1960:1–1998:4, 1957:1–1980:2, 1957:1–1998:1, 1946:1–2001:2 for Canada, France, West Germany before reunification, Germany after unification, Italy, Japan United Kingdom and United States, respectively. The data for expenditures and revenues –as a share of the GDP– of the central governments of the G-7 countries are shown in Figure 4.2. The series appear to have a nonstationary behavior, as assumed in the discussion of the previous sections.

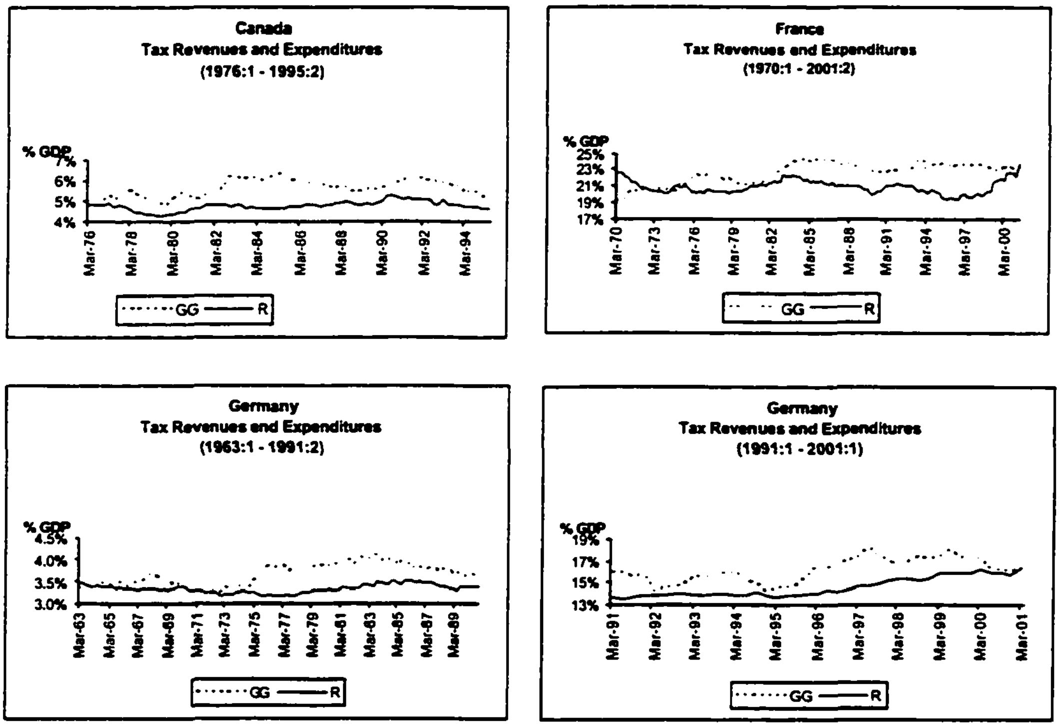
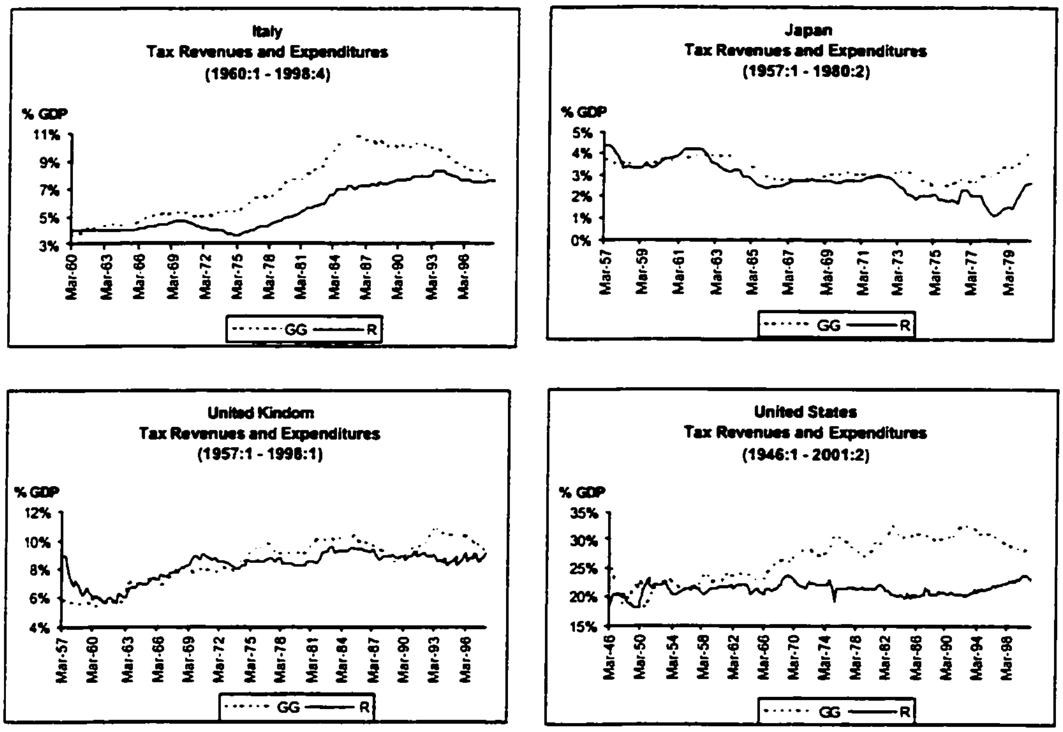


Figure 4.2. Revenues and Expenditures of the Central Government for the G7 Countries (Quarterly data as % of GDP)

As mentioned in previous sections, to assess the sustainability of a fiscal policy we need to test for cointegration between expenditures and revenues. Before doing that, it is needed to test the order of integration of the revenues and expenditures. To investigate the number of unit roots present in each series the Augmented Dickey Fuller (1979) –ADF– and Phillips-Perron (1989) –PP– tests will be used.



Italy
Tax Revenues and Expenditures
(1960:1 - 1998:4)

Japan
Tax Revenues and Expenditures
(1957:1 - 1980:2)

United Kindom
Tax Revenues and Expenditures
(1957:1 - 1998:1)

United States
Tax Revenues and Expenditures
(1946:1 - 2001:2)

Figure 4.2. (Cont.) Revenues and Expenditures of the Central Government for the G7 Countries (Quarterly data as % of GDP)

Table 4.1. Unit Root Tests on the Revenues and Expenditures.

Country	Test		Levels		first differences	
			GG	R	GG	R
Canada		τ	0.0134	-0.3908	-3.5233 ***	-2.8928 ***
(1976:1 - 1995:2)	ADF	τ_μ	-3.5034 ***	-2.3973	-3.4363 ***	-2.8755 **
		τ_τ	-2.4782	-2.9995	-3.7743 **	-3.8602 ***
		τ	0.2279	-0.3413	-8.7987 ***	-9.2409 ***
	PP	τ_μ	-2.2202 **	-1.7386	-8.7561 ***	-9.1886 ***
		τ_τ	-1.6455	-1.9405	-8.9814 ***	-9.1347 ***
France		τ	0.4119	0.6317	-3.7416 ***	-3.1444 ***
(1970:1 - 2001:2)	ADF	τ_μ	-1.9112	-2.1816	-3.7395 ***	-3.1391 **
		τ_τ	-2.4194	-2.0223	-3.7180 **	-3.3442 **
		τ	1.1844	0.1820	-12.6199 ***	-10.0163 ***
	PP	τ_μ	-2.6476 *	-1.6813	-12.7640 ***	-9.9756 ***
		τ_τ	-2.2882	-1.2432	-12.9757 ***	-10.3221 ***
Germany		τ	0.2379	0.1063	-3.6246 ***	-3.3680 ***
(1963:1 - 1990:4)	ADF	τ_μ	-1.5910	-1.6983	-3.6228 ***	-3.3437 **
		τ_τ	-2.8086	-2.1320	-3.6084 **	-3.4066 *
		τ	0.4454	-0.3985	-10.2477 ***	-12.0800 ***
	PP	τ_μ	-1.5857	-1.7505	-10.2266 ***	-12.0373 ***
		τ_τ	-1.4869	-2.1656	-10.2340 ***	-12.1824 ***
Germany		τ	0.4839	0.2119	-3.3967 ***	-1.8128 *
(1991:1 - 2001:1)	ADF	τ_μ	-2.3036	0.2082	-3.3634 **	-2.9483 **
		τ_τ	-2.4244	-2.1686	-3.4247 *	-5.1759 ***
		τ	-0.0600	0.7138	-6.3888 ***	-5.3272 ***
	PP	τ_μ	-1.6952	0.7058	-6.3120 ***	-6.0161 ***
		τ_τ	-2.2847	-1.6775	-6.2339 ***	-6.0827 ***

Note: 1) The regression equations used to test for the presence of a unit root are: $\Delta y_t = \psi y_{t-1} + \sum \varphi_{t+1+i} \Delta y_{t+1+i} + \varepsilon_t$ (a), $\Delta y_t = \alpha_0 + \psi y_{t-1} + \sum \varphi_{t+1+i} \Delta y_{t+1+i} + \varepsilon_t$, (b), and $\Delta y_t = \alpha_0 + \psi y_{t-1} + \alpha_1 t + \sum \varphi_{t+1+i} \Delta y_{t+1+i} + \varepsilon_t$ (c).

2) The statistics labeled as τ, τ_μ, and τ_τ are the corresponding statistics to use for equations (a), (b), and (c), respectively.

3) The number of lags for the ADF test was selected by minimizing the AIC. The lags for the PP test were set according to the suggestions of Newey and West (1994).

4) *, **, and *** denote the rejection of the null hypothesis of a unit root at the 10%, 5%, and 1% levels, respectively.

Table 4.1. (Cont.) Unit Root Tests on the Revenues and Expenditures.

Country	Test		Levels				first differences			
			GG		R		GG		R	
Italy		τ	-0.4584		1.1112		-1.6544	**	-3.4761	***
(1960:1 - 1998:4)	ADF	τ_μ	-1.5810		-0.7198		-2.5530	**	-3.7342	***
		τ_τ	-0.5900		-1.5491		-3.5042	**	-3.7089	**
		τ	0.9374		2.2399		-9.0234	***	-8.8875	***
	PP	τ_μ	-1.6424		-0.2905		-9.1749	***	-9.4216	***
		τ_τ	1.2887		-1.2908		-9.6436	***	-9.3973	***
Japan		τ	0.6256		-0.7955		-1.8606	*	-4.2222	***
(1957:1 - 1980:2)	ADF	τ_μ	-1.3685		-1.5149		-1.9470	**	-4.1855	***
		τ_τ	-0.4173		-2.6781		-3.6232	**	-4.1896	***
		τ	0.2931		-1.3828		-6.0555		-6.6630	***
	PP	τ_μ	-0.7942		-2.2020		-6.0275		-6.6861	***
		τ_τ	0.9723		-2.3651		-6.4943		-6.8051	***
United Kingdom		τ	0.0134		0.7997		-7.5767	***	-3.5484	***
(1957:1 - 1998:1)	ADF	τ_μ	-1.8052		-2.6609	*	-7.6487	***	-3.6711	***
		τ_τ	-1.8308		-2.7363		-7.7334	***	-3.7921	**
		τ	0.9454		-0.0830		-13.2614	***	-13.9410	***
	PP	τ_μ	-1.6187		-1.4784		-13.3492	***	-13.9000	***
		τ_τ	-1.6484		-3.3060	*	-13.3973	***	-13.9257	***
U.S.		τ	0.6238		0.3738		-4.0844	***	-9.8905	***
(1946:1 - 2001:2)	ADF	τ_μ	-1.3057		-3.2104	**	-4.3255	***	-9.8743	***
		τ_τ	-1.3669		-3.1268		-4.7084	***	-9.8568	***
		τ	-0.0853		0.5488		-12.9554	***	-15.7482	***
	PP	τ_μ	-1.1442		-2.1776	**	-12.9315	***	-15.7368	***
		τ_τ	-3.5918	**	-2.6054		-12.8921	***	-15.6991	***

Note: 1) The regression equations used to test for the presence of a unit root are:
$\Delta y_t = \psi y_{t-1} + \sum \varphi_{t+1+i} \Delta y_{t+1+i} + \varepsilon_t$ (a), $\Delta y_t = \alpha_0 + \psi y_{t-1} + \sum \varphi_{t+1+i} \Delta y_{t+1+i} + \varepsilon_t$, (b), and
$\Delta y_t = \alpha_0 + \psi y_{t-1} + \alpha_1 t + \sum \varphi_{t+1+i} \Delta y_{t+1+i} + \varepsilon_t$ (c).

2) The statistics labeled as τ, τ_μ, and τ_τ are the corresponding statistics to use for equations (a), (b), and (c), respectively.

3) The number of lags for the ADF test was selected by minimizing the AIC. The lags for the PP test were set according to the suggestions of Newey and West (1994).

4) *, **, and *** denote the rejection of the null hypothesis of a unit root at the 10%, 5%, and 1% levels, respectively.

root at the 10%, 5%, and 1% levels, respectively.

According to the results of the unit root tests, reported in Table 4.1, there is strong evidence to accept the null of a presence of a unit root in the levels for all the series and all 8 samples. There is also enough evidence to reject the null of unit roots in the first differences of the series. Thus, we conclude that expenditures and revenues are I(1) for all 7 countries.

4.1.1 Testing for Cointegration

According to Hakkio and Rush (1991), a necessary condition for a fiscal policy to be sustainable is that revenues and taxes (expressed as a share of the GDP) are cointegrated, with cointegrating vector [1,-*b*], with *b*=1; although $0 < b < 1$ guarantees the fulfillment of the intertemporal budget constraint. As mentioned in Chapter 3, if the interest in cointegration is motivated by the possibility of a particular known cointegrating vector, then we may use this information using this value directly to construct a test of cointegration.

Thus with a known cointegrating vector, $b = 1$, we can test for sustainability of the fiscal policy by defining $D_t \equiv R_t - GG_t$ and testing if this series is I(0) or not. The results of this test are reported in Table 4.2[34]. According to these tests, only the United Kingdom's deficit seems to be stationary. From this we can conclude that the U.K.'s revenues and expenditures are linearly cointegrated with cointegrating vector [1,-1] and thus according to this test the U.K.'s fiscal policy is sustainable. For the rest of the countries we cannot reject the null hypothesis of the existence of one unit root in the deficit series and hence fiscal policies are not sustainable for these countries.

In Table 4.2 the Horvath and Watson (1995) test -HW- is also reported a multivariate test that takes advantage of a known cointegrating vector.[35] The results of this test suggest cointegration between revenues and expenditures for Canada, France, Germany, Japan, the U.K., and the U.S..

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[34] Note that according to our discussion there is not need to run the ADF test with a deterministic trend.

[35] For details on this test see

Table 4.2. Cointegration Test with known Cointegrating Vector [1,-1].

Country		ADF Statistic	Lags		PP Test Statistic	Lags	HW Statistic
Canada	τ	-0.516	(1)	τ	-0.341	(3)	7.558 *
(1976:1 - 1995:2)	τ_μ	-2.478	(1)	τ_μ	-1.739	(3)	
France	τ	-1.144	(5)	τ	-1.736 *	(4)	13.061 ***
(1970:1 - 2001:2)	τ_μ	-2.042	(5)	τ_μ	-3.252 **	(4)	
Germany	τ	-0.656	(5)	τ	-0.723	(4)	11.375 **
(1963:1 - 1990:4)	τ_μ	-1.788	(5)	τ_μ	-2.091	(4)	
Germany	τ	-1.282	(1)	τ	-1.453	(3)	6.750 *
(1991:1 - 2001:1)	τ_μ	-1.950	(1)	τ_μ	-1.964	(3)	
Italy	τ	-0.636	(5)	τ	0.937	(4)	6.102
(1960:1 - 1998:4)	τ_μ	-0.685	(5)	τ_μ	-1.642	(4)	
Japan	τ	0.400	(5)	τ	-0.858	(3)	7.843 *
(1957:1 - 1980:2)	τ_μ	-0.475	(5)	τ_μ	-1.810	(3)	
United Kingdom	τ	-2.088 **	(8)	τ	-3.400 ***	(4)	6.764 *
(1957:1 - 1998:1)	τ_μ	-2.360	(8)	τ_μ	-3.786 ***	(4)	
U.S.	τ	-0.256	(8)	τ	-1.426	(4)	7.805 *
(1946:1 - 2001:2)	τ_μ	-1.303	(8)	τ_μ	-2.055	(4)	

Note: 1) The regression equations used to test for the presence of a unit root in the deficit are:
$\Delta D_t = \psi D_{t-1} + \sum \varphi_{t+1+i} \Delta D_{t+1+i} + \varepsilon_t$ (a) and $\Delta D_t = \alpha_0 + \psi D_{t-1} + \sum \varphi_{t+1+i} \Delta D_{t+1+i} + \varepsilon_t$ (b).

2) The statistics labeled as ? and ?? are the corresponding statistics to use for equations (a) and (b), respectively.

3) The number of lags for the ADF test was selected by minimizing the AIC. The lags for the PP test were set according to the suggestions of Newey and West (1994) (1994). The number of lags of the HW test is selected by minimizing a multivariate version of the AIC

4) The Horvath and Watson test critical values at the 1%, 5%, and 10% are 12.18, 8.47, and 6.63, respectively

5) *, **, and *** denote the rejection of the null hypothesis of no cointegration at the 10%, 5%, and 1% levels, respectively.

As suggested by the Monte Carlo study reported in Chapter 3, we also perform other linear cointegration tests that do not assume a cointegration vector such as the Engle-Granger and Johansen tests. The Engle-Granger test – See Table 4.4– leads us to conclude that revenues and expenditures are cointegrated and thus a "strict" interpretation of the intertemporal budget constraint is satisfied. Note that the point estimates for the slope are relatively small and in all cases less than one. However, Engle and Granger's (1987) method has been shown to have low power for testing the null hypothesis of a cointegrating vector $[1,-b]$ (with $0 < b \leq 1$), because standard least squares estimators provide biased estimators of the regression coefficients.

Table 4.3.Engle-Granger Cointegration Test.

	$R_t = a + bGG_t + \varepsilon_t$				
	Estimates		ADF		
Country	a	b	Stat.		Lags
Canada	0.0336	0.2503	-18.519	***	(1)
(1976:1 - 1995:2)	(0.0030)	(0.0537)			
France	0.2128	-0.0127	-22.356	***	(4)
(1970:1 - 2001:2)	(0.0127)	(0.0561)			
Germany	0.0291	0.1148	-37.162	***	(4)
(1963:1 - 1990:4)	(0.0012)	(0.0336)			
Germany	0.0593	0.5401	-9.791	***	(1)
(1991:1 - 2001:1)	(0.0156)	(0.0962)			
Italy	0.0103	0.641	-125.232	***	(5)
(1960:1 - 1998:4)	(0.0017)	(0.0225)			
Japan	-0.0101	1.186	-24.166	***	(5)
(1957:1 - 1980:2)	(0.0044)	(0.1361)			
United Kingdom	0.0379	0.5274	-34.362	***	(8)
(1957:1 - 1998:1)	(0.0023)	(0.0271)			
U.S.	0.2112	0.0085	-26.259	***	(2)
(1946:1 - 2001:2)	(0.0049)	(0.0183)			

Note: 1) The number of lags for the ADF test was selected by minimizing the AIC.
2) Standard errors in parenthesis.
3) The Engle and Yoo (1987) critical values were used.
4) *, **, and *** denote the rejection of the null hypothesis of no cointegration at the 10%, 5%, and 1% levels, respectively.

According to Johansen's cointegration test, see Table 4.4, France and the U.S. are the only countries that exhibit cointegration between the expenditures and revenues. The normalized estimated cointegrating vectors are [1, -0.485] and [1, -0.085] for France and the

U.S., respectively. A likelihood ratio test[36] for the null hypothesis of a cointegrating vector [1,-1] may be used; under the null this test follows a χ^2 distribution with one degree of freedom[37]. The Likelihood ratio test statistics are 23.4 and 16.8 for France and the U.S., respectively. In both cases the null hypothesis of $0 < b \leq 1$ is accepted and the null hypothesis of $b = 1$ is rejected. Thus, according to the Johansen cointegration tests, it may conclude that the American and French fiscal policies for the respective study period are satisfying a "strict" interpretation of the Intertemporal Budget Constraint. However, these structures of expenditures and revenues are not sustainable, since this estimated cointegrating vector implies a debt to GDP ratio going to infinity although the PVBC is satisfied.

For the case of Italy and Germany before 1990, the Johansen test provides evidence of the existence of two cointegrating vectors. This result implies that both the expenditures and revenues may be stationary, but we have already found evidence in favor of the non-stationarity of those series. The rest of the countries do not present cointegration between revenues and expenditures.

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[36] The Likelihood ratio test statistic is given by $T\sum_{i=1}^{r}\left[\ln\left(1-\hat{\lambda}_i^{UR}\right)-\ln\left(1-\hat{\lambda}_i^{R}\right)\right]$, where r is the number of cointegrating vectors. $\hat{\lambda}_i^{UR}$ and $\hat{\lambda}_i^{R}$ denote the ordered characteristic roots of the unrestricted and restricted models.

[37] Number of cointegrating vectors.

Table 4.4. Johansen Maximum Likelihood Ratio Cointegration Test

Country	H_o	H_a	λ_{trace}		H_o	H_a	λ_{max}	
Canada	r=0	r>0	10.548		r=0	r=1	8.402	
(1976:1 - 1995:2)	r<=1	r>1	2.146		r=1	r=2	2.146	
France	r=0	r>0	20.289	***	r=0	r=1	20.252	***
(1970:1 - 2001:2)	r<=1	r>1	0.038		r=1	r=2	0.038	
Germany	r=0	r>0	15.806	**	r=0	r=1	11.514	
(1963:1 - 1990:4)	r<=1	r>1	4.291	**	r=1	r=2	4.291	
Germany	r=0	r>0	7.621		r=0	r=1	7.419	
(1991:1 - 2001:1)	r<=1	r>1	0.201		r=1	r=2	0.201	**
Italy	r=0	r>0	25.467	***	r=0	r=1	19.16	***
(1960:1 - 1998:4)	r<=1	r>1	6.307	**	r=1	r=2	6.307	**
Japan	r=0	r>0	7.011		r=0	r=1	6.97	
(1957:1 - 1980:2)	r<=1	r>1	0.041		r=1	r=2	0.041	
United Kingdom	r=0	r>0	11.847		r=0	r=1	7.808	
(1957:1 - 1998:1)	r<=1	r>1	4.039	**	r=1	r=2	4.039	**
U.S.A.	r=0	r>0	13.961	*	r=0	r=1	12.385	*
(1946:1 - 2001:2)	r<=1	r>1	1.576		r=1	r=2	1.576	

Note: 1) The number of lags in the test was assigned by selecting the lag length of the VAR in levels by minimizing the AIC.
2) *, **, and *** denote the rejection of the null hypothesis of a unit root at the 10%, 5%, and 1% levels, respectively.

Note that the results from the linear cointegration tests are contradictory. However this result should not be that surprising given the results of the Monte Carlo study of Chapter 3. To summarize the result so far, the ADF test on the residuals of the cointegrating equation with known cointegrating vector lead us to conclude that the U.K. and France have sustainable fiscal policies. The HW test with known cointegrating vector provides evidence in favor of cointegration of the revenues and expenses for all the G-7 countries, except Italy. With the Engle and Granger test, it is concluded that the residuals of the estimated cointegrating equation are stationary for all 7 countries. According to the Johansen cointegration test, it is concluded that the U.S. and France have sustainable fiscal policy.

As suggested in Chapter 3, Enders and Granger (1998) and Berben and van Dijk (1999) – BVD – cointegration tests, whish are designed for non-linear processes under the alternative hypothesis, are applied to our problem. The results of the Enders and Granger test – see Table 4.5 – permit us to reject the null of no-cointegration for all the G-7 countries except the U.S., Italy, and Germany for the sample period before the reunification. On the other hand, using the BVD tests the null hypothesis of no-cointegration is rejected only for Canada and the U.S., for the rest of countries there is no evidence in favor of cointegration.

Putting together all the evidence from the cointegration tests, we may conclude that the Italian government does not present a sustainable fiscal policy for the period of study – 1960:1 to 1998:4–. For the rest of the countries there exists some evidence in favor of cointegration. As mentioned in Chapter 3, according to our Monte Carlo experiment there is not a single cointegration tests that poses more power than the others considered. The power of the test depends on the true data generating process, which is of course unknown in the practice. In Chapter 3 we also found numerous cases for which all but one cointegration tests rejected the null of no-cointegration even though the true system was cointegrated. Given those results and the results of our cointegration tests for the total revenues and expenditures of the G-7 countries, we may claim that there is some evidence in favor of the sustainability of the fiscal policies in Canada, France, Germany, Japan, the U.K. and the U.S..

Table 4.5. Enders and Granger (EndG) and Berben and van Dijk (BVD) tests for (Threshold) Cointegration

Country	EndG Stat	BVD Stat	Country	EndG Stat	BVD Stat
Canada (1976:1 - 1995:2)	4.397 ***	5.436 **	Italy (1960:1 - 1998:4)	0.492	0.900
France (1970:1 - 2001:2)	5.758 ***	3.048	Japan (1957:1 - 1980:2)	2.442 **	2.099
Germany (1963:1 - 1990:4)	1.592	2.836	United Kingdom (1957:1 - 1998:1)	4.907 ***	2.920
Germany (1991:1 - 2001:1)	5.960 ***	1.967	U.S.A. (1946:1 - 2001:2)	1.227	4.990 **

Note: 1) The number of lags in each of the test was assigned by selecting the lag length of that minimizes the AIC. For the HW a multivariate version of the AIC was used.

2) The BVD test critical values at the 1%, 5%, and 10% are 5.57, 4.35, and 3.71, respectively.

3) *, **, and *** implies rejection of the null hypothesis of no cointegration at 10%, 5% and 1%significant level, respectively.

4.1.2 Estimating a TVECM for Total Expenditures and Revenues of the G-7's Central governments

Once it has been determined that the revenues and expenditures are cointegrated (linearly or non-linearly), we may proceed to find out what type of short-run adjustment desccribes each of the systems. In other words, now it is possible to investigate if a VECM or a TVECM better fits the data, i.e., a general two or three-regime TVECM, a Band-TVECM, or an Equilibrium-TVECM.

In this particular case, a three-regime TVECM is given by

$$\begin{bmatrix} \Delta R_t \\ \Delta GG_t \end{bmatrix}_t = \begin{cases} \gamma^{(3)}\left(\left(R_{t-1} - bGG_{t-1}\right) - \mu^{(3)}\right) + \sum_{i=1}^{p-1} \theta_{1,i}^{(3)} \Delta R_{t-i} + \sum_{i=1}^{p-1} \theta_{2,i}^{(3)} \Delta GG_{t-i} + \varepsilon_t, \text{ if } & z_{t-d} > c^{(2)} \\ \gamma^{(2)}\left(\left(R_{t-1} - bGG_{t-1}\right) - \mu^{(2)}\right) + \sum_{i=1}^{p-1} \theta_{1,i}^{(2)} \Delta R_{t-i} + \sum_{i=1}^{p-1} \theta_{2,i}^{(2)} \Delta GG_{t-i} + \varepsilon_t, \text{ if } & c^{(1)} \le z_{t-d} \le c^{(2)} \\ \gamma^{(1)}\left(\left(R_{t-1} - bGG_{t-1}\right) - \mu^{(1)}\right) + \sum_{i=1}^{p-1} \theta_{1,i}^{(1)} \Delta R_{t-i} + \sum_{i=1}^{p-1} \theta_{2,i}^{(1)} \Delta GG_{t-i} + \varepsilon_t, \text{ if } & z_{t-d} < c^{(1)} \end{cases} \tag{4.1}$$

where d is the delay parameter, R_t are the central Government's total revenues, and GG_t are the central Government's total expenditures inclusive of debt service. If $\gamma^{(2)} = 0$, then (3.3) is known as a Band-TVECM. Furthermore, if in addition $\mu^{(1)} = \mu^{(3)} = 0$, then (3.3) is known as an EQ-TVECM[38].

In this section, we will follow closely the technique outlined in section 3.3 to find the best VECM for the different G-7 countries. As mentioned before, the first step is to find out if a non-linear model describes the data better than a linear model. Let the linear VECM be model 1, the 2-regime TVECM be model 2, the unrestricted three-regime TVECM be model 3, the Band TVECM be model 4, and the EQ TVECM be model 5.

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[38] See Chapter 3 for more details on the particular models.

Table 4.6. Model Specification Likelihood Ratio Test.

Country	Ho:	VECM (1)		VECM (1)		2TVECM (2)	
	Ha:	2TVECM (2)		3TVECM (3)		3TVECM (3)	
		LR_{12}	P-val.	LR_{13}	P-val.	LR_{23}	P-val.
Canada	p= 1	6.6262 **	0.016	18.69079 **	0.037	12.06 **	0.045
(1976:1 - 1995:2)	p= 2	5.1288 **	0.026	15.81446 *	0.079	10.69 **	0.048
	p= 3	12.498 **	0.026	15.21325 *	0.051	2.716 **	0.045
	p= 4	8.23 *	0.077	22.22517 *	0.075	14.18 **	0.047
	p= 5	9.0365 **	0.019	19.54909 **	0.046	10.51 **	0.033
	p=6	8.1864 *	0.081	30.20467 *	0.081	22.02 **	0.024
France	p= 1	17.879 **	0.031	22.44317 **	0.042	4.564 **	0.048
(1970:1 - 2001:2)	p= 2	31.751 ***	0.007	33.98906 **	0.018	2.238 **	0.047
	p= 3	1.7822 *	0.069	17.16715 *	0.066	15.38 **	0.049
	p= 4	2.1067 *	0.067	17.5096 *	0.060	15.53 **	0.027
	p= 5	11.545 *	0.062	1.928527 *	0.058	350 ***	0.001
	p=6	3.6659 **	0.049	13.54281 *	0.052	9.877 **	0.026
Germany	p= 1	4.4765 **	0.030	11.01263 *	0.080	6.536 **	0.047
(1963:1 - 1990:4)	p= 2	6.6541 *	0.078	8.758392 *	0.078	15.41 **	0.044
	p= 3	1.2488 *	0.070	12.7053 *	0.067	11.46 **	0.029
	p= 4	11.263 ***	0.005	18.5166 *	0.064	7.32 **	0.029
	p= 5	11.211 **	0.022	20.008 **	0.021	860.2 ***	0.000
	p=6	8.9877 *	0.066	13.26125 *	0.061	33.48 **	0.042
Germany	p= 1	7.3316 **	0.014	20.68052 **	0.03272	13.35 **	0.0397
(1991:1 - 2001:1)	p= 2	5.6748 **	0.023	17.49798 *	0.07021	11.82 **	0.0426
	p= 3	13.828 **	0.023	16.83277 **	0.04518	3.005 **	0.0401
	p= 4	9.1061 *	0.069	24.59115 *	0.06715	15.69 **	0.0421
	p= 5	9.9985 **	0.017	21.63018 **	0.04065	11.63 **	0.0293
	p=6	9.0579 *	0.072	33.4201 *	0.07182	24.36 **	0.0211

Note: 1) *, **, and *** implies rejection of the relevant null hypothesis at 10%, 5% and 1%significant level, respectively.

The likelihood ratio test and sup-Wald tests described in section 3.3.2 may be used to test the null hypothesis of a linear model against the alternative of a non-linear model (model 2 or model 3)[39]. Table 4.6 presents the *LR* statistics and the corresponding simulated p-values for the linearity tests. The likelihood ratio statistics were calculated for $p = 1,\ldots,6$. In all the cases, the linear model is rejected in favor of a two or three-regime TVECM. Table 4.6 also presents the likelihood ratio statistics and corresponding p-values for testing the null hypothesis of a 2-regime versus a three-regime TVECM. Again, for all the cases the null hypothesis of a two-regime TVECM is rejected. The same conclusions are drawn if the sup-F tests are used – these results are reported in Appendix 2–.

Table 4.6. (Cont.) Model Specification Likelihood Ratio Test.

Country	Ho:	VECM (1)		VECM (1)		2TVECM (2)	
	Ha:	2TVECM (2)		3TVECM (3)		3TVECM (3)	
		LR_{12}	P-val.	LR_{13}	P-val.	LR_{23}	P-val.
Japan	p= 1	19.783 **	0.0273	24.83235 **	0.03758	5.05 **	0.0428
(1957:1 - 1980:2)	p= 2	35.131 ***	0.0058	37.60736 **	0.01609	2.476 **	0.0422
	p= 3	1.9719 *	0.0618	18.99468 *	0.05912	17.02 **	0.0435
	p= 4	2.3309 *	0.0599	19.37358 *	0.05298	17.18 **	0.0236
	p= 5	12.774 *	0.0551	2.133827 *	0.05158	387.2 ***	0.0009
	p=6	4.0561 **	0.0434	14.98451 **	0.04625	10.93 **	0.0233
United Kingdom	p= 1	4.953 **	0.0271	12.18497 *	0.07098	7.232 **	0.0419
(1957:1 - 1998:1)	p= 2	7.3625 *	0.0692	9.690765 *	0.06959	17.05 **	0.0389
	p= 3	1.3817 *	0.0627	14.05784 *	0.05958	12.68 **	0.0257
	p= 4	12.462 ***	0.0048	20.48778 *	0.05695	8.099 **	0.0262
	p= 5	12.404 **	0.0196	22.13795 **	0.01905	951.7 ***	0.0003
	p=6	9.9445 *	0.0584	14.67297 *	0.05438	37.04 **	0.0371
U.S.	p= 1	8.1121 **	0.0124	22.88206 **	0.02912	14.77 **	0.0353
(1946:1 - 2001:2)	p= 2	6.2789 **	0.0209	19.36073 *	0.06249	13.08 **	0.0379
	p= 3	15.3 **	0.0209	18.6247 **	0.04021	3.325 **	0.0357
	p= 4	10.075 *	0.0613	27.20899 *	0.05976	17.37 **	0.0374
	p= 5	11.063 **	0.015	23.93282 **	0.03618	12.87 **	0.0261
	p=6	10.022 *	0.0638	36.97783 *	0.06392	26.96 **	0.0188

Note: 1) *, **, and *** implies rejection of the relevant null hypothesis at 10%, 5% and 1%significant level, respectively.

If we consider the TVECM and the Band-TVECM, for all the countries and lag periods, we may conclude that the Band-TVECM fits the data sets better. On the other hand, if we test the null of an Equilibrium-TVECM against the alternative of a TVECM, then we cannot reject the null that an Equilibrium-TVECM fits the data for all countries and lag lengths. These results are reported in Appendix 2.

The Likelihood Ratio test and the sup-Wald test for the null hypothesis of the Equilibrium-TVECM versus the alternative of a Band-TVECM do not give us enough

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[39] The corresponding Likelihood Ratio statistics are LR_{12}, LR_{13}.. for details on the tests see Chapter 3.

evidence to reject the null for all the countries – See Appendix 2. Thus, we may conclude that the Equilibrium-TVECM explains the data as well as the Band-TVECM for all the G-7 countries with sustainable fiscal policies.

Table 4.7. The Equilibrium TVECM for R_t and GG_t with known cointegrating vector [1,-1]

Country		Regime 1		Regime 2		Regime 3	
		ΔR_t	ΔGG_t	ΔR_t	ΔGG_t	ΔR_t	ΔGG_t
Canada	$R_{t-1} - GG_{t-1}$	-0.0373 ***	0.0227	—	—	-0.1004 ***	0.0014
(1976:1 - 1995:2)		(0.0132)	(0.0417)			(0.0343)	(0.0331)
(p=2, d=7)	ΔR_{t-1}	-0.5674 ***	-0.1615 ***	-0.0787 ***	-0.1715 ***	-0.0874	-0.1902 **
		(0.2142)	(0.0207)	(0.0231)	(0.0223)	(0.0939)	(0.0906)
	ΔR_{t-2}	-0.2981	-0.1688 ***	-0.0771	0.1669	-0.0950 ***	0.0224 ***
		(0.2338)	(0.0226)	(0.2404)	(0.2321)	(0.0195)	(0.0016)
	ΔGG_{t-1}	-0.0103	-0.1703	-0.0318 *	-0.0242	0.0716	0.0672 ***
		(0.3970)	(0.3833)	(0.0186)	(0.0179)	(0.1041)	(0.0101)
	ΔGG_{t-2}	0.2326	0.5809 ***	0.0880 ***	0.1255 ***	0.1979 *	0.1759 *
		(0.4290)	(0.0414)	(0.0165)	(0.0159)	(0.1053)	(0.1017)
		$z_{t-7} < -1.1673$		$-1.1673 < z_{t-7} < -.5225$		$z_{t-7} > -.5225$	
		$T_1 = 21$		$T_2 = 100$		$T_3 = 43$	
France	$R_{t-1} - GG_{t-1}$	-0.0066	0.0248 **	—	—	-0.1056 ***	0.0529
(1970:1 - 2001:2)		(0.0142)	(0.0104)			(0.0349)	(0.0348)
(p=1, d=3)							
		$z_{t-3} < -2.4587$		$-2.4587 < z_{t-3} < .7031$		$z_{t-3} > .7031$	
		$T_1 = 40$		$T_2 = 50$		$T_3 = 35$	
Germany	$R_{t-1} - GG_{t-1}$	-0.0207 ***	-0.0312 *	—	—	0.0397	-0.2448 **
(1963:1 - 1990:4)		(0.0074)	(0.0179)			(0.0426)	(0.1028)
(p=3, d=4)	ΔR_{t-1}	-0.3444 **	-0.1465	-0.1868	-0.2114	0.1953	-0.2638
		(0.1645)	(0.3968)	(0.1418)	(0.3421)	(0.2033)	(0.4905)
	ΔR_{t-2}	-0.3079 *	-0.6040	-0.0646	-0.6872 **	0.0991	-1.0434 **
		(0.1649)	(0.3979)	(0.1336)	(0.3224)	(0.1831)	(0.4418)
	ΔGG_{t-1}	0.1988 ***	-0.3735 **	0.1593 **	0.1312	0.1390 *	-0.1907
		(0.0655)	(0.1581)	(0.0670)	(0.1616)	(0.0832)	(0.2008)
	ΔGG_{t-2}	-0.0057	-0.2986 *	-0.1617 **	-0.2509 *	-0.1557 *	-0.1205
		(0.0721)	(0.1740)	(0.0621)	(0.1498)	(0.0833)	(0.2009)
		$z_{t-3} < -.5299$		$-.5299 < z_{t-3} < -.1617$		$z_{t-3} > -.1617$	
		$T_1 = 34$		$T_2 = 35$		$T_3 = 40$	

Note: 1) Standard errors in parenthesis

2) p is the AR order and d is the delay parameter.

3) z_{t-7} is the threshold variable.

4) *, **, and *** implies a significantly different than zero coefficient at 10%, 5% and 1%significant level, respectively.

After identifying that the EQ-TVECM is the model that best describes the data, we need to choose the lag length that best describes the data. To select the lag length of the Equilibrium TVECM, we use a multivariate version of the AIC and SBC statistics[40]. The selected models are reported in Table 4.7.

Table 4.7. (Cont.) The Equilibrium TVECM for R_t and GG_t with known cointegrating vector [1,-1]

Country		Regime 1		Regime 2		Regime 3	
		ΔR_t	ΔGG_t	ΔR_t	ΔGG_t	ΔR_t	ΔGG_t
Germany (1991:1 - 2001:1) *(p=1, d=4)*	$R_{t-1} - GG_{t-1}$	-0.0375 (0.0680)	0.3710 ** (0.1609)	—	—	-0.0570 * (0.0337)	0.0890 *** (0.0080)
		$z_{t-3} < -2.0321$ $T_1 = 8$		$-2.0321 < z_{t-3} < -.5322$ $T_2 = 17$		$z_{t-3} > -.5322$ $T_3 = 15$	
Japan (1957:1 - 1980:2) *(p=1, d=4)*	$R_{t-1} - GG_{t-1}$	-0.0974 *** (0.0324)	-0.0556 *** (0.0201)	—	—	-0.0475 ** (0.0239)	-0.0608 *** (0.0158)
		$z_{t-3} < -.6456$ $T_1 = 18$		$-.6456 < z_{t-3} < -.1102$ $T_2 = 45$		$z_{t-3} > -.1102$ $T_3 = 30$	
United Kingdom (1957:1 - 1998:1) *(p=1, d=3)*		-0.0248 *** (0.0018)	0.0238 * (0.0127)	—	—	-0.1574 *** (0.0393)	0.0237 (0.0373)
		$z_{t-3} < -.8695$ $T_1 = 35$		$-.8695 < z_{t-3} < .2174$ $T_2 = 78$		$z_{t-3} > .2174$ $T_3 = 51$	

Note: 1) Standard errors in parenthesis
2) p is the AR order and d is the delay parameter.
3) z_{t-7} is the threshold variable.
4) *, **, and *** implies a significantly different than zero coefficient at 10%, 5% and 1%significant level, respectively.

For Canada, the estimated model implies that the fiscal authority reacts with a delay of 7 periods – one year and 9 months – to short-run disequilibria that are "big" or "small" enough. The estimated thresholds for the Canadian fiscal policy are deficits of 1.167% of the GDP and 0.5225% of the GDP. When a fiscal deficit is bigger than the threshold deficit of 1.167% of the GDP (Regime 1), an adjustment to the long-run equilibrium is made with a 7-quarter delay. The revenues are increased by 0.037% of the GDP per percentage point in the deficit;

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[40] For more details see Chapter 3.

at the same time there is not a change in the expenditures[41], so that the system may return to the long-run equilibrium path. On the other hand, if the deficit is smaller than 0.5225% of the GDP (Regime 3), then the reaction is to reduce the revenues by 0.1% of the GDP per percentage point in the deficit; while there is no change in the expenditures.

According to our estimated model, Japan's fiscal authorities present a different behavior. When the fiscal deficit is above 0.65% of the GDP, then both revenues and expenditures are increased, but in different proportions: the revenues are increased by 0.097% of GDP and the revenues by 0.055% of GDP per percentage point of deficit. When the deficit is low enough, i.e., lower than 0.11%, then the expenditures are increased by a bigger amount than the increase in the revenues.

The U.S. presents a different behavior too. For deficits larger than 5.78% of the GDP, the adjustment is made by increasing the revenues in 0.046% per percentage point of deficit. For deficits lower than 2.05% of the GDP, expenditures are increased by 0.09%.

The estimated models may also be used to identify periods for which the fiscal policy was outside the thresholds. In Chapter 1 and Chapter 2, a sustainable fiscal policy was defined as a situation for which the current structure of the central government's revenues and expenditures could be maintained without violating the intertemporal budget constraint. As seen in Chapter 2, this definition of a sustainable fiscal policy implies a global behavior of the time series corresponding to the revenues and expenditures. These two series have to cointegrate with cointegrating vector [1,-1]. However, TVECMs allow for a local situation in which no long-run relation is present, but still globally the series present a long-run relationship. For instance, whenever in an EQ-TVECM the short-run disequilibria are outside the thresholds, an adjustment to the long-run relationship is made.

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[41] Note that the estimate for $\gamma^{(1)}$ in (3.3) is not significantly different from zero.

Table 4.7. The Equilibrium TVECM for R_t and GG_t with known cointegrating vector [1,-1]

Country		Regime 1		Regime 2		Regime 3	
		ΔR_t	ΔGG_t	ΔR_t	ΔGG_t	ΔR_t	ΔGG_t
U.S.A.	$R_{t-1} - GG_{t-1}$	-0.0460 ***	0.0022	—	—	-0.0070	-0.0907 **
(1946:1 - 2001:2)		(0.0045)	(0.0055)			(0.0421)	(0.0417)
(p=6, d=4)	ΔR_{t-1}	-0.3311 ***	-0.2200	-0.0920	-0.6184 ***	0.1355	-0.1845
		(0.1259)	(0.1543)	(0.1425)	(0.1747)	(0.1483)	(0.1817)
	ΔR_{t-2}	-0.0985	-0.2459 *	0.5290 ***	0.1138	-0.2127 ***	0.1501 ***
		(0.1213)	(0.1486)	(0.2014)	(0.2469)	(0.0135)	(0.0166)
	ΔR_{t-3}	-0.0847	0.0424	-0.5189 ***	0.6769 ***	0.1163	0.0305
		(0.1117)	(0.1369)	(0.1943)	(0.2381)	(0.1336)	(0.1638)
	ΔR_{t-4}	-0.0523 ***	-0.0171	-0.1275	-0.1142	-0.0367 **	-0.2545
		(0.0112)	(0.1376)	(0.1760)	(0.2158)	(0.0142)	(0.1745)
	ΔR_{t-5}	-0.0805	-0.0546	0.2304	-0.1455	0.0346 ***	0.1972
		(0.1011)	(0.1240)	(0.1703)	(0.2088)	(0.0128)	(0.1570)
	ΔGG_{t-1}	-0.0665	0.2597 *	-0.1584 *	0.0836	-0.1070	-0.0095
		(0.1252)	(0.1534)	(0.0889)	(0.1090)	(0.0965)	(0.1183)
	ΔGG_{t-2}	-0.0871	0.1106	-0.1351	-0.1331	0.0599	0.3448 ***
		(0.1196)	(0.1466)	(0.0868)	(0.1064)	(0.0998)	(0.1224)
	ΔGG_{t-3}	0.0193 *	0.2275 *	-0.1401	0.2333 **	0.0880	0.1575
		(0.0110)	(0.1342)	(0.0892)	(0.1093)	(0.0797)	(0.0977)
	ΔGG_{t-4}	0.0590 ***	-0.1789	0.3093 ***	0.2179 *	0.1325 *	0.0130
		(0.0117)	(0.1433)	(0.0935)	(0.1146)	(0.0779)	(0.0955)
	ΔGG_{t-5}	-0.0719	0.0089	0.1569	-0.6162 ***	-0.0914	0.0262
		(0.1133)	(0.1389)	(0.1188)	(0.1457)	(0.0697)	(0.0854)
		$z_{t-4} < -6.7852$		$-6.7852 < z_{t-4} < -2.0556$		$z_{t-4} > -2.0556$	
		$T_1 = 100$		$T_2 = 58$		$T_3 = 60$	

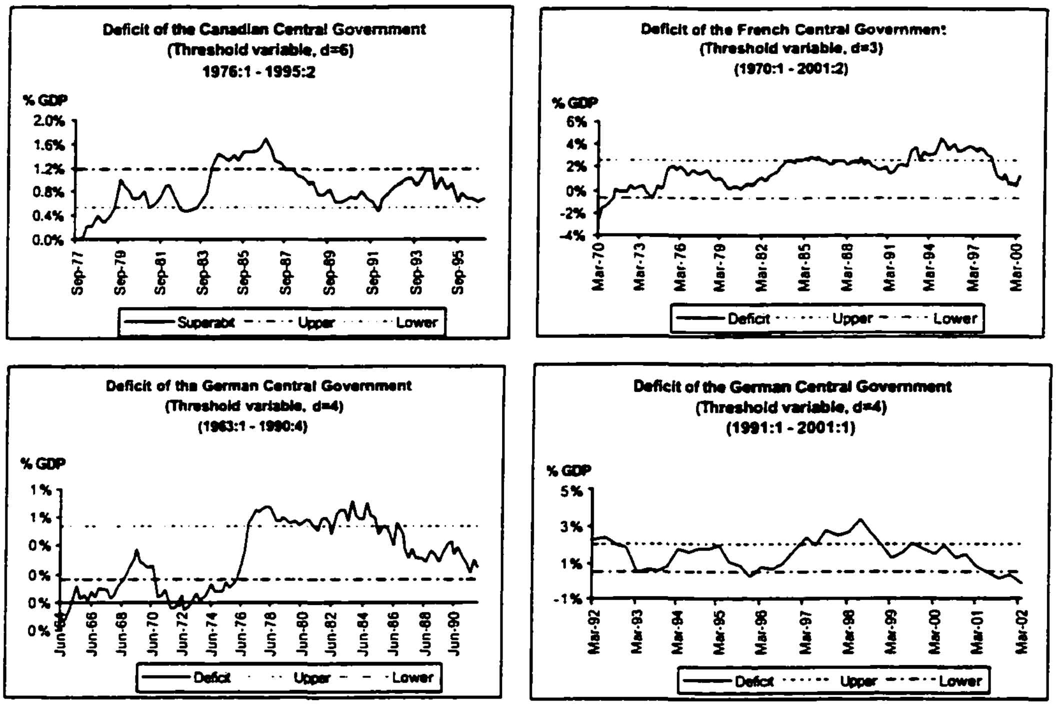
Note: 1) Standard errors in parenthesis

2) p is the AR order and d is the delay parameter.

3) z_{t-7} is the threshold variable.

4) *, **, and *** implies a significantly different than zero coefficient at 10%, 5% and 1%significant level, respectively.

Thus, we may interpret the episodes for which the system is outside the thresholds of an EQ-TVECM as situations where adjustments were made to achieve the long-run relationship. And thus those episodes may be considered as "unsustainable" situations, since adjustments to the fiscal policy were made to bring the fiscal policy back to a long-run sustainable path, although the whole system presents a globally sustainable behavior. Or in other words, periods for which the long-run relationship was "turned on", and according to our model action was taken to bring the system to its long-run relationship, will be considered as "unsustainable" periods. On the other hand, when the long-run relationship is "turned off", no adjustment is needed; and thus the period corresponds to a sustainable episode. Periods for which the deficit is too “low” – below the smallest threshold in absolute values– will not be considered as “unsustainable periods. If the deficit is becoming smaller, then the intertemporal budget constraint is clearly being fulfilled.


Deficit of the Canadian Central Government
(Threshold variable, d=6)
1976:1 - 1995:2

Deficit of the French Central Government
(Threshold variable, d=3)
(1970:1 - 2001:2)

Deficit of tha German Central Government
(Threshold variable, d=4)
(1963:1 - 1990:4)

Deficit of the German Central Government
(Threshold variable, d=4)
(1991:1 - 2001:1)

Figure 4.3. Threshold Variable.

For example, for Canada the estimating model suggests that the deficit was above the largest threshold from the 1st quarter of 1984 to the 2nd quarter of 1987, reaching its historical maximum deficit for the period by the 3rd quarter of 1986 –See Figure 4.3. Note that previously Smith and Zin (1991) found, using monthly data and linear methods, for the period 1946–84 that the Canadian fiscal policy was not sustainable. On the other hand, Payne (1997) using annual data for the period 1949-1993 found evidence in favor of a sustainable Canadian fiscal policy. Our estimated model reconciles these two results. As mentioned before, for the mid 1980s the estimated model suggests a period for which the deficit was "big" enough so that the long run relationship between revenues and expenditures began to work. This period, or at least the period after the maximum is reached, may be considered as a period of adjusting to the long run equilibrium path. This result could be the effect captured in Smith and Zin's (1991) unsustainable result. On the other hand, for the late 1980s and the 1990s the model presents a fiscal situation where the deficit is under control and no adjustment needs to be done, which according to our definition implies a sustainable situation, a result that coincides with Payne's (1997) sustainable result.

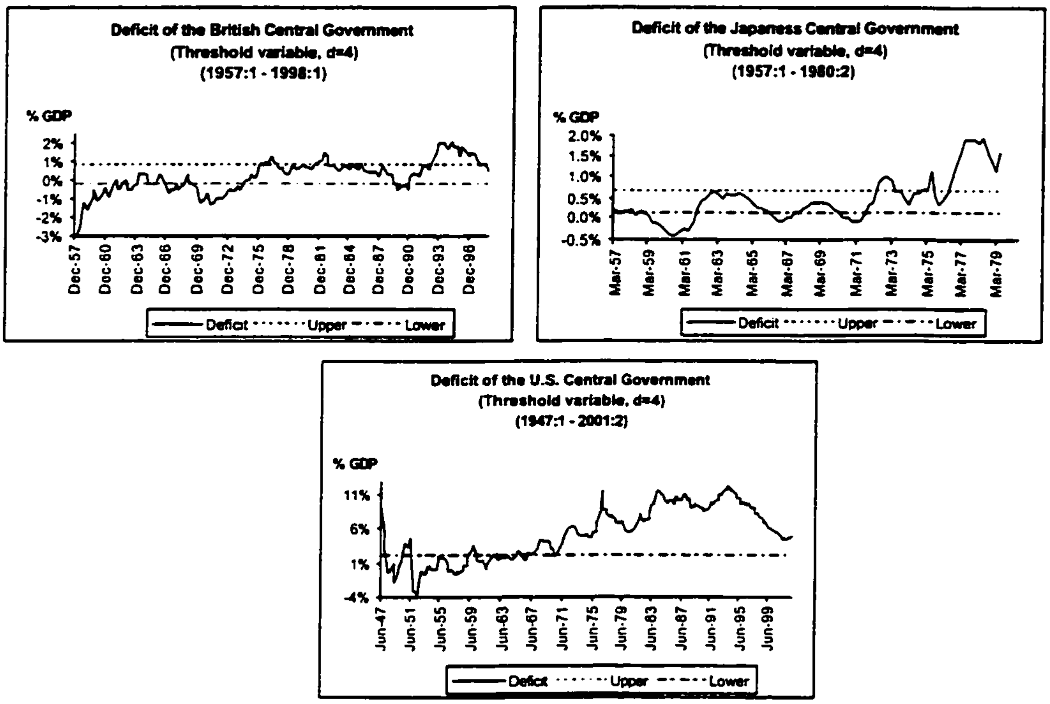


Figure 4.3. (Cont.) Threshold Variable.

For the U.S., the estimated model suggests a long period during the 1980s and most of the 1990s[42] for which the fiscal deficit is above the upper threshold. The maximum federal quarterly deficit – 12.6% of the GDP – is reached by the 1st quarter of 1994 and from then on

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[42] From the 1981's 3rd quarter to 1999's 1st quarter.

the deficit began its tendency back to the long-run sustainable path. Previously, Hakkio and Rush (1991) found evidence for the unsustainability of the U.S. fiscal policy using quarterly data for the period 1950–1988. According to our estimated model, the end of Hakkio and Rush's sample period coincides to a situation were adjustment need to be done, and thus a linear cointegration approach may be capturing this unsustainable situation of the end of the period. Payne (1997) using annual data for the period 1949–1994 coincides in his findings with Hakkio and Rush' unsustainability conclusion for the U.S. fiscal policy. Again, the end of Payne's sample period corresponds to a period where, according to our model, an adjustment to the long run relation ship was needed. In fact according to the graph, the adjustment, or at least the change in the tendency of the deficit, had not begun by the end of 1994.

Feve and Henin (2000) found evidence in favor of sustainability of the U.S. fiscal policy using annual data for the period 1956-1999. In this case, Feve and Henin's sample ends at a period where a long-run path is achieved, or at least is on the way. This may explain the coincidence of Feve and Henin's sustainable result and our model's findings.

In a similar fashion, the models for the other countries may be used to identify "unsustainable" situations. Other "unsustainable" periods identified are: 1992:3 to 1998:2 for France, 1976:3 to 1980:2 for Japan, and 1993:1 and 1997:3 for the U.K.. Thus, in this section we found that the overall structure of revenues and expenditures for Canada, France, Germany, Japan, the U.K, and the U.S. are sustainable for the whole sample. However, the fiscal policies for those countries present periods for which the long-run relationship between revenues and expenditures – including debt service – of the central governments switches "on" and "off". In other words, the short-run dynamics present a non-linear behavior that permitted us to identify "unsustainable" episodes.

The non-linear behavior of the adjustment seems very intuitive, when one considers all the political costs associated with any adjustment in the fiscal structure. Such costs do not permit an instant adjustment towards the long-run equilibrium, so the adjustment is done only when a threshold is reached. We were able to find evidence in favor of the later argument. Furthermore, we estimated the thresholds for which the fiscal authority did not take any action to bring the system back to the long-run equilibrium. In the rest of this chapter we will study the sustainability of the current account of the G-7 countries, and investigate the nature of the short-run dynamics. The next subsection will study the sustainability of the current account of the G-7 countries.

4.2 Sustainability of the Current Account for the G-7 Countries

In this subsection we investigate the sustainability of the current account of the G-7 countries. As in the previous section, the data used in this section come from the International Financial Statistics data base published by the IMF. In the case of the trade variables the IMF data set is more complete than in the case of the fiscal data. The particular samples used in the following analysis are: for Canada, the U.K., and the U.S. from 1957:1 to 2001:4, for France from 1965:1 to 2001:4, for Germany from 1971:1 to 2001:3, and for Italy from 1965:1 to 2001:4. The data for total exports (X_t) and total imports minus net interest payments (MM_t) as a share of the GDP of the G-7 countries are shown in Figure 4.4.

Husted (1992) showed that a sustainable current account implies that total exports and total imports plus net interest payments and net transfer payments should be cointegrated with cointegrating vector [1,-1], if X_t and MM_t are I(1) and are expressed as a share of the GDP. To confirm the non-stationarity of the series X_t and MM_t for the G-7 countries the ADF and PP tests are performed to both the level and first differences of the series.

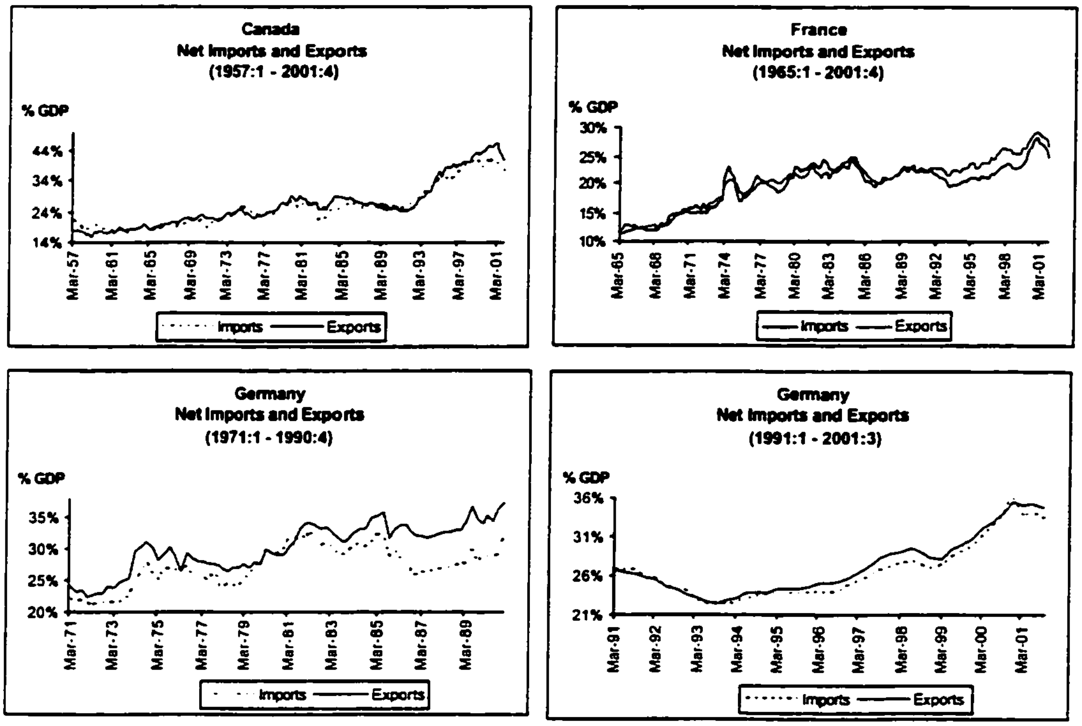

Figure 4.4. Total Imports and Exports of the G7 Countries (Quarterly data as % of GDP)

According to the results of the ADF and PP tests –reported in Table 4.8– the series are I(1) for all the 7 countries. In the next subsections we will tests for sustainability and estimate the VECM –linear or non-linear– that better fits the data in case cointegration among the variables is found.

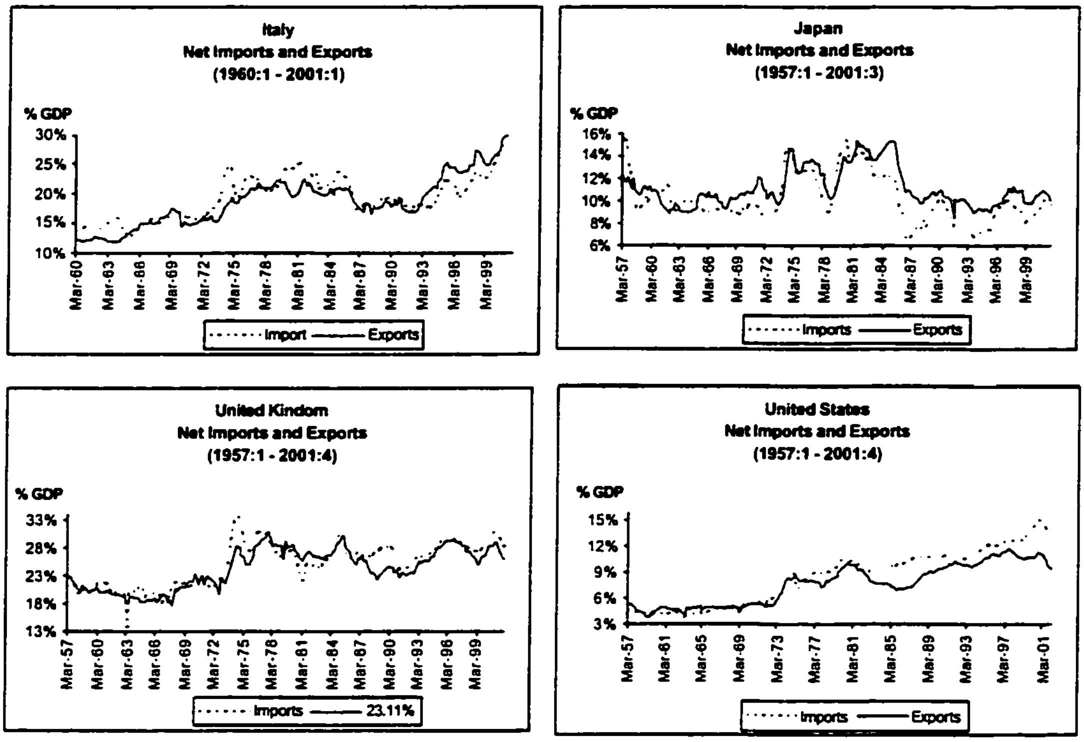

Figure 4.4. Total Imports and Exports of the G7 Countries

(Quarterly data as % of GDP)

Table 4.8. Unit Root Tests on the Exports and Imports

Country	Test		Levels			first differences			
			M		X	M		X	
Canada		τ	1.3546		1.0651	-7.8685	***	-4.9123	***
(1976:1 - 1995:2)	ADF	τ_μ	-0.3156-		0.7264	-7.9861	***	-5.0645	***
		τ_τ	-2.2264-		2.2640	-7.9904	***	-5.0021	***
		τ	1.3699		1.8852	-12.5937	***	-13.6232	***
	PP	τ_μ	-0.0063		0.1054	-12.7005	***	-13.8800	***
		τ_τ	-2.4302-		1.7595	-12.7567	***	-13.9042	***
France		τ	1.1332		1.4205	-7.2500	***	-6.8164	***
(1970:1 - 2001:2)	ADF	τ_μ	-1.7396-		1.3766	-7.3708	***	-6.9453	***
		τ_τ	-2.1783-		2.3709	-7.3675	***	-6.9319	***
		τ	0.8031		1.4850	-8.2049	***	-10.6439	***
	PP	τ_μ	-2.0313-		1.4756	-8.2326	***	-10.8244	***
		τ_τ	-2.5572-		2.5236	-8.2356	***	-10.8111	***
Germany		τ	0.9370		1.2256	-5.6832	***	-6.7571	***
(1974:1 - 1990:4)	ADF	τ_μ	-1.6306-		1.3149	-5.7405	***	-6.8986	***
		τ_τ	-1.9205-		2.7785	-5.6884	***	-6.8572	***
		τ	0.9029		1.2560	-9.2302	***	-9.4972	***
	PP	τ_μ	-1.5608-		1.0792	-9.3114	***	-9.7177	***
		τ_τ	-2.0583-		2.9507	-9.2519	***	-9.6497	***
Germany		τ	0.9391		1.1236	-2.7626	***	-2.2728	**
(1991:1 - 2001:1)	ADF	τ_μ	1.1426		0.0690	-3.0398	**	-4.4795	**
		τ_τ	-3.3645	*	-2.8126	-4.2228	***	-4.5901	***
		τ	1.0509		1.4746	-5.1931	***	-3.4418	***
	PP	τ_μ	0.1882		0.6983	-5.2913	***	-3.6603	***
		τ_τ	-1.9479-		2.1152	-5.8970	***	-4.1884	**

Note: 1) The regression equations used to test for the presence of a unit root are:

$\Delta y_t = \psi y_{t-1} + \sum \varphi_{t+1+i} \Delta y_{t+1+i} + \varepsilon_t$ (a), $\Delta y_t = \alpha_0 + \psi y_{t-1} + \sum \varphi_{t+1+i} \Delta y_{t+1+i} + \varepsilon_t$, (b), and

$\Delta y_t = \alpha_0 + \psi y_{t-1} + \alpha_1 t + \sum \varphi_{t+1+i} \Delta y_{t+1+i} + \varepsilon_t$ (c).

2) The statistics labeled as τ, τ_μ, and τ_τ are the corresponding statistics to use for equations (a), (b), and (c), respectively.

3) The number of lags for the ADF test was selected by minimizing the AIC. The lags for the PP test were set according to the suggestions of Newey and West (1994).

4) *, **, and *** denote the rejection of the null hypothesis of a unit root at the 10%, 5%, and 1% levels, respectively.

Table 4.8. Unit Root Tests on the Exports and Imports. (Cont.)

Country	Test		Levels			first differences			
			M		X	M		X	
Italy (1960:1 - 2001:3)	ADF	τ	1.0408		1.8148	-7.7267	***	-8.0775	***
		τ_μ	-1.0901		-0.1746	-7.8035	***	-8.2417	***
		τ_τ	-1.7329		-1.4486	-7.7907	***	-8.2636	***
	PP	τ	0.8671		1.7830	-9.8136	***	-12.0964	***
		τ_μ	-1.3675		-0.1956	-9.8476	***	-12.2991	***
		τ_τ	-2.1162		-1.5911	-9.8271	***	-12.3114	***
Japan (1957:1 - 2001:4)	ADF	τ	-0.5587		-0.4167	-7.4695	***	-7.2437	***
		τ_μ	-2.4922		-2.1569	-7.4464	***	-7.2231	***
		τ_τ	-2.6052		-2.1407	-7.4059	***	-7.2022	***
	PP	τ	-1.1864		-0.6309	-7.9898	***	-10.8167	***
		τ_μ	-3.4069	**	-2.4158	-7.9895	***	-10.7893	***
		τ_τ	-3.3172		-2.4116	-8.0038	***	-10.7582	***
United Kingdom (1957:1 - 2001:4)	ADF	τ	0.3298		0.1408	-7.4870	***	-8.4625	***
		τ_μ	-2.0886		-1.8002	-7.4857	***	-8.4455	***
		τ_τ	-3.3359	*	-2.3496	-7.4613	***	-8.4189	***
	PP	τ	-0.0249		-0.0017	-12.6309	***	-14.5033	***
		τ_μ	-2.1828		-1.6916	-12.6030	***	-14.4723	***
		τ_τ	-2.5248		-2.5130	-12.5664	***	-14.4335	***
U.S. (1957:1 - 2001:4)	ADF	τ	1.4090		0.1632	-8.2988	***	-4.6777	***
		τ_μ	-0.7269		-1.7739	-8.4394	***	-4.7174	***
		τ_τ	-2.9548		-2.7894	-8.4059	***	-4.6908	***
	PP	τ	1.4266		0.5381	-12.6297	***	-12.4000	***
		τ_μ	-0.6544		-1.0067	-12.7994	***	-12.4295	***
		τ_τ	-3.0796		-2.4970	-12.7629	***	-12.3999	***

Note: 1) The regression equations used to test for the presence of a unit root are:

$\Delta y_t = \psi y_{t-1} + \sum \varphi_{t+1+i} \Delta y_{t+1+i} + \varepsilon_t$ (a), $\Delta y_t = \alpha_0 + \psi y_{t-1} + \sum \varphi_{t+1+i} \Delta y_{t+1+i} + \varepsilon_t$, (b), and

$\Delta y_t = \alpha_0 + \psi y_{t-1} + \alpha_1 t + \sum \varphi_{t+1+i} \Delta y_{t+1+i} + \varepsilon_t$ (c).

2) The statistics labeled as τ, τ_μ , and τ_τ are the corresponding statistics to use for equations (a), (b), and (c), respectively.

3) The number of lags for the ADF test was selected by minimizing the AIC. The lags for the PP test were set according to the suggestions of Newey and West (1994).

4) *, **, and *** denote the rejection of the null hypothesis of a unit root at the 10%, 5%, and 1% levels, respectively.

4.2.1 Testing for Cointegration

Following the same procedure as in section 4.1.1, we test for cointegration both using the information of the known cointegrating vector and estimating the cointegrating vector. Using the known cointegrating vector $[1,-1]$, we may construct the series $z_t = X_t - MM_t$, and then test with ADF and PP tests whether the constructed series is stationary or not. We also performed the HW multivariate cointegration test using the known cointegrating vector.

Table 4.9. Cointegration Test with known Cointegrating Vector [1,-1].

Country		ADF Statistic	Lags		PP Test Statistic	Lags	HW Statistic
Canada	τ	-1.915 *	(1)	τ	-2.32 **	(4)	9.1758 **
(1976:1 - 1995:2)	τ_μ	-2.54	(1)	τ_μ	-2.97 **	(4)	
France	τ	-2.23 **	(1)	τ	-2.28 **	(4)	7.6928 **
(1970:1 - 2001:2)	τ_μ	-2.59 *	(1)	τ_μ	-2.72 *	(4)	
Germany	τ	-0.04	(1)	τ	-0.38	(3)	4.0157
(1974:1 - 1990:4)	τ_μ	-1.19	(1)	τ_μ	-1.65	(3)	
Germany	τ	-0.4	(1)	τ	-0.64	(3)	1.5338
(1991:1 - 2001:1)	τ_μ	-1.87	(1)	τ_μ	-1.96	(3)	
Italy	τ	-2.75 ***	(1)	τ	-2.57 **	(4)	4.9905 *
(1960:1 - 2001:3)	τ_μ	-2.73 *	(1)	τ_μ	-2.56	(4)	
Japan	τ	-2.61 **	(2)	τ	-2.95 ***	(4)	34.928 ***
(1957:1 - 2001:4)	τ_μ	-3.54 ***	(3)	τ_μ	-4.18 ***	(4)	
United Kingdom	τ	-2.93 ***	(1)	τ	-3.46 ***	(4)	19.647 ***
(1957:1 - 2001:4)	τ_μ	-3.19 **	(1)	τ_μ	-3.76 ***	(4)	
U.S.	τ	-0.71	(1)	τ	-0.77	(4)	8.9632 *
(1957:1 - 2001:4)	τ_μ	-1.49	(1)	τ_μ	-1.57	(4)	

Note: 1) The regression equations used to test for the presence of a unit root in the deficit are: $\Delta D_t = \psi D_{t-1} + \sum \varphi_{t+1+i} \Delta D_{t+1+i} + \varepsilon_t$ (a) and $\Delta D_t = \alpha_0 + \psi D_{t-1} + \sum \varphi_{t+1+i} \Delta D_{t+1+i} + \varepsilon_t$ (b).

2) The statistics labeled as τ and τ_μ are the corresponding statistics to use for equations (a) and (b), respectively.

3) The number of lags for the ADF test was selected by minimizing the AIC. The lags for the PP test were set according to the suggestions of Newey and West (1994). The number of lags of the HW test is selected by minimizing a multivariate version of the AIC

4) The Horvath and Watson test critical values at the 1%, 5%, and 10% are 12.18, 8.47, and 6.63, respectively

5) *, **, and *** denote the rejection of the null hypothesis of no cointegration at the 10%, 5%, and 1% levels, respectively.

The cointegration tests with known cointegrating vector, reported in Table 4.9., provide mixed conclusions regarding the sustainability of the U.S. current account. The ADF and PP suggest an unsustainable current account for the U.S., meanwhile the HW provides evidence in favor of the sustainability of the U.S. current account. For the rest of the countries the three tests coincide in their conclusions. The tests suggest an unsustainable current account for Germany. For the rest of the countries the tests provide evidence in favor of cointegration between X_t and MM_t, with cointegrating vector $[1,-1]$.

Table 4.10. Engle-Granger Cointegration Test.

	$R_t = a + bGG_t + \varepsilon_t$			
	Estimates		ADF	
Country	a	b	Stat.	Lags
Canada	-0.0274	1.1457	-25.4524 ***	(2)
(1976:1 - 1995:2)	(0.0048)	(0.0185)		
France	0.0017	1.0244	-26.0846 ***	(1)
(1970:1 - 2001:2)	(0.0053)	(0.0257)		
Germany	0.025	1.022	-18.466 ***	(1)
(1974:1 - 1990:4)	(0.0195)	(0.0716)		
Germany	-0.003	1.0361	-9.11595 ***	(1)
(1991:1 - 2001:1)	(0.0070)	(0.0259)		
Italy	0.0018	0.9683	-37.9458 ***	(1)
(1960:1 - 2001:3)	(0.0088)	(0.0448)		
Japan	0.0489	0.6242	-33.4595 ***	(3)
(1957:1 - 2001:4)	(0.0040)	(0.0392)		
United Kingdom	0.0372	0.8269	-24.7129 ***	(1)
(1957:1 - 2001:4)	(0.0077)	(0.0306)		
U.S.	0.0174	0.7007	-39.673 ***	(3)
(1957:1 - 2001:4)	(0.0016)	(0.0171)		

Note: 1) The number of lags for the ADF test was selected by minimizing the AIC.
2) Standard errors in parenthesis.
3) The Engle and Yoo (1987) critical values were used.
4) *, **, and *** denote the rejection of the null hypothesis of no cointegration at the 10%, 5%, and 1% levels, respectively.

On the other hand, the Engel and Granger cointegration test –see table 4.10 – suggests cointegration between X_t and MM_t for all the countries. However the biased estimates for the normalized cointegrating vector are extremely low. Clearly, this is not a formal test for the cointegrating vector equal to $[1,-1]$. The null hypothesis of a cointegrating vector $[1,-1]$ is more naturally tested in the multivariate setup given by Johansen's test.

Table 4.11. Johansen Maximum Likelihood Ratio Cointegration Test

Country	H_o	H_a	λ_{trace}		H_o	H_a	λ_{max}	
Canada	r=0	r>0	15.712	**	r=0	r=1	15.679	**
(1976:1 - 1995:2)	r<=1	r>1	0.033		r=1	r=2	0.033	
France	r=0	r>0	13.497		r=0	r=1	11.944	
(1970:1 - 2001:2)	r<=1	r>1	1.553		r=1	r=2	1.553	
Germany	r=0	r>0	4.195		r=0	r=1	3.117	
(1974:1 - 1990:4)	r<=1	r>1	1.077		r=1	r=2	1.077	
Germany	r=0	r>0	5.137		r=0	r=1	5.118	
(1991:1 - 2001:1)	r<=1	r>1	0.019		r=1	r=2	0.019	
Italy	r=0	r>0	10.854		r=0	r=1	10.819	
(1960:1 - 2001:3)	r<=1	r>1	0.035		r=1	r=2	0.035	
Japan	r=0	r>0	27.072	***	r=0	r=1	19.663	***
(1957:1 - 2001:4)	r<=1	r>1	7.409	***	r=1	r=2	7.409	***
United Kingdom	r=0	r>0	14.907	*	r=0	r=1	12.342	
(1957:1 - 2001:4)	r<=1	r>1	2.565		r=1	r=2	2.565	
U.S.	r=0	r>0	5.463		r=0	r=1	4.983	
(1957:1 - 2001:4)	r<=1	r>1	0.480		r=1	r=2	0.480	

Note: 1) The number of lags in the test was assigned by selecting the lag length of the VAR in levels by minimizing the AIC.

2) *, **, and *** denote the rejection of the null hypothesis of a unit root at the 10%, 5%, and 1% levels, respectively.

Johansen's tests for cointegration between X_t and MM_t is reported in table 4.11. This test suggests that the U.K. and Canada present cointegration. The normalized estimated cointegrating vectors are [1, -1.06] and [1,-1.03] for Canada and the U.K., respectively. A likelihood ratio test[43] for the null hypothesis of a cointegrating vector [1,-1] may be used;

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[43] The Likelihood ratio test statistic is given by $T\sum_{i=1}^{r}\left[\ln\left(1-\hat{\lambda}_i^{UR}\right)-\ln\left(1-\hat{\lambda}_i^{R}\right)\right]$, where r is the number of cointegrating vectors. $\hat{\lambda}_i^{UR}$ and $\hat{\lambda}_i^{R}$ denote the ordered characteristic roots of the unrestricted and restricted models.

under the null this test follows a χ^2 distribution with one degree of freedom[44]. The Likelihood ratio test statistics are 1258.3 and 1176.2 for Canada and the U.K., respectively. In both cases the null hypothesis of $b=1$ may not be rejected. Thus, according to the Johansen cointegration tests, the U.K. and Canada present sustainable current accounts.

For the case of Japan, the Johansen test provides evidence of the existence of two cointegrating vectors. This result implies that both the expenditures and revenues may be stationary and therefore not cointegrated, but we have already found evidence in favor of the non-stationarity of those series.

We also applied the Enders and Granger and the BVD stationarity test on the constructed cointegrating error $z_t = X_t - MM_t$. The results of these two tests – see Table 4.12 – imply that the Italian and post-reunification German current accounts are not sustainable.

Table 4.12. Enders and Granger (EndG) and Berben and van Dijk (BVD) tests for (Threshold) Cointegration

Country	EndG Stat	BVD Stat	Country	EndG Stat	BVD Stat
Canada (1976:1 - 1995:2)	4.699 **	5.4361 **	Italy (1960:1 - 2001:3)	1.4288	3.5284
France (1970:1 - 2001:2)	4.2002 *	6.2225 ***	Japan (1957:1 - 2001:4)	10.166 ***	15.431 ***
Germany (1974:1 - 1990:4)	1.1257	1.6515	United Kingdom (1957:1 - 2001:4)	8.5214 ***	6.8979 ***
Germany (1991:1 - 2001:1)	3.5934	8.5033 ***	U.S.A. (1957:1 - 2001:4)	2.0043	7.184 ***

Note: 1) The number of lags in each of the test was assigned by selecting the lag length of that minimizes the AIC. For the HW a multivariate version of the AIC was used.

2) The BVD test critical values at the 1%, 5%, and 10% are 5.57, 4.35, and 3.71, respectively.

3) *, **, and *** implies rejection of the null hypothesis of no cointegration at 10%, 5% and 1%significant level, respectively.

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[44] Number of cointegrating vectors.

The results of the cointegration tests on X_t and MM_t are mixed. All the cointegration tests and the test on the cointegrating vector lead us to conclude that for Canada and the U.K. the current account is sustainable. For the U.S. the results of the cointegration tests are mixed. HW, BVD, and Engle and Granger test give evidence in favor of the cointegration of X_t and MM_t; the first two tests imply a cointegrating vector [1,-1] while the Engle and Granger do not. Evidence in favor of the unsustainability of the U.S. current account is given by the ADF, PP, and Enders and Granger test on the cointegrating residual and the Johansen Test. For the case of the U.S., we will conclude that the series are in fact cointegrated, since in the remaining of the chapter we will see that the possibility of non-linearity is ruled out, and Horvath and Watson(1995) showed that the HW test has more power than the ADF and PP tests on the cointegrating residuals and the Johansen test.

Using the same argument as above, we will conclude that the current accounts for France, Italy and Japan are sustainable, while the German is not.

4.2.2 Estimating The Short-Run dynamics of Total Exports and Imports for the G-7 Countries

After we have determined the cointegration of X_t and MM_t, and thus the sustainability of the current account, we may investigate the short-run behavior of the cointegrated system. The first step is to find out if a non-linear model explains better than a linear model the short run dynamics. The unrestricted three-regime TVECM for this case is given by

$$\begin{bmatrix} \Delta X_t \\ \Delta MM_t \end{bmatrix}_t = \begin{cases} \gamma^{(3)}\left((X_{t-1} - bMM_{t-1}) - \mu^{(3)}\right) + \sum_{i=1}^{p-1} \theta_{1,i}^{(3)} \Delta X_{t-i} + \sum_{i=1}^{p-1} \theta_{2,i}^{(3)} \Delta MM_{t-i} + \varepsilon_t, \text{ if } & z_{t-d} > c^{(2)} \\ \gamma^{(2)}\left((X_{t-1} - bMM_{t-1}) - \mu^{(2)}\right) + \sum_{i=1}^{p-1} \theta_{1,i}^{(2)} \Delta X_{t-i} + \sum_{i=1}^{p-1} \theta_{2,i}^{(2)} \Delta MM_{t-i} + \varepsilon_t, \text{ if } & c^{(1)} \leq z_{t-d} \leq c^{(2)} \\ \gamma^{(1)}\left((X_{t-1} - bMM_{t-1}) - \mu^{(1)}\right) + \sum_{i=1}^{p-1} \theta_{1,i}^{(1)} \Delta X_{t-i} + \sum_{i=1}^{p-1} \theta_{2,i}^{(1)} \Delta MM_{t-i} + \varepsilon_t, \text{ if } & z_{t-d} < c^{(1)} \end{cases} \tag{4.2}$$

and the linear VECM is given by

$$\begin{bmatrix} \Delta X_t \\ \Delta MM_t \end{bmatrix} = \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix} + \begin{bmatrix} \gamma_1 (X_{t-1} - bMM_{t-1}) \\ \gamma_2 (X_{t-1} - bMM_{t-1}) \end{bmatrix} + \sum_{i=1}^{p-1} \Psi_i \begin{bmatrix} \Delta X_{t-i} \\ \Delta MM_{t-i} \end{bmatrix} + \varepsilon_t . \tag{4.3}$$

As discussed in Chapter 3, Hansen's (1999) *LR* and sup-Wald tests may be used as a linearity test. The *LR* linearity test, reported in table 4.13., does not reject the null hypothesis of a linear VECM, when the alternative model is either a two-regime or a three-regime TVECM. The sup-Wald test, reported in Appendix 2 leads to the same conclusion. Therefore non-linearity may be rejected for all the G-7 countries with sustainable current accounts. In other words, adjustments to any short-run disequilibrium are symmetric and have the same magnitude no matter the sign of the disequilibria.

Table 4.13. Model Specification Likelihood Ratio Test.

Country	Ho:	VECM (1)		VECM (1)		2TVECM (2)	
	Ha:	2TVECM (2)		3TVECM (3)		3TVECM (3)	
		LR_{12}	P-val.	LR_{13}	P-val.	LR_{23}	P-val.
Canada	p= 0	22.545	0.605	31.861	0.8549.3	16	0.855
(1957:1 - 2001:4)	p= 1	18.816	0.718	51.522	0.721	21.047	0.844
	p= 2	25.362	0.709	40.683	0.8879.9	40	0.896
	p= 3	29.160	0.552	68.965	0.653	33.458	0.587
	p= 4	36.432	0.787	75.746	0.816	38.900	0.594
	p= 5	34.863	0.497	87.683	0.447	39.500	0.485
France	p= 0	29.260	0.282	36.431	0.3127.1	71	0.408
(1965:1 - 2001:4)	p= 1	28.057	0.690	51.315	0.953	22.702	0.838
	p= 2	42.626	0.618	81.641	0.565	36.546	0.184
	p= 3	44.156	0.839	86.837	0.892	37.491	0.674
	p= 4	44.894	0.622	94.738	0.940	29.938	0.750
	p= 5	47.353 *	0.073	103.636	0.856	60.808	0.485
Italy	p= 0	25.202	0.407	45.992	0.593	20.789	0.763
(1960:1 - 2001:1)	p= 1	30.956	0.582	54.633	0.708	23.670	0.804
	p= 2	29.241	0.795	61.193	0.746	24.679	0.856
	p= 3	13.250	0.767	34.759	0.480	13.914	0.630
	p= 4	24.059	0.482	44.249	0.266	17.565	0.404
	p= 5	28.228	0.647	57.710	0.136	25.061	0.203

Note: 1) *, **, and *** implies rejection of the relevant null hypothesis at 10%, 5% and 1%significant level, respectively.

Table 4.13. Model Specification Likelihood Ratio Test. (Cont.)

Country	Ho: Ha:	VECM (1) 2TVECM (2)		VECM (1) 3TVECM (3)		2TVECM (2) 3TVECM (3)	
		LR_{12}	P-val.	LR_{13}	P-val.	LR_{23}	P-val.
Japan	p= 0	92.375	0.223	106.098	0.204	13.723	0.307
(1957:1 - 2001:3)	p= 1	76.687	0.307	105.387	0.680	28.700	0.805
	p= 2	67.556	0.350	85.575	0.989	18.018	0.850
	p= 3	51.624	0.586	100.295	0.990	40.110	0.394
	p= 4	58.592	0.828	112.874	1.000	47.319 ***	0.007
	p= 5	56.194	0.122	135.667	0.994	51.203	0.393
U.K.	p= 0	38.762	0.661	39.695	0.766	0.932	0.965
(1957:1 - 2001:4)	p= 1	34.908	0.710	86.479	0.389	37.391	0.613
	p= 2	38.601	0.695	92.725	0.412	43.116	0.624
	p= 3	41.335	0.775	98.851	0.521	40.962	0.793
	p= 4	47.608	0.919	108.672	0.486	49.382	0.611
	p= 5	46.382	0.989	122.042	0.666	60.733	0.721
U.S.	p= 0	9.426	0.898	16.005	0.860	6.579	0.709
(1957:1 - 2001:4)	p= 1	32.563	0.537	55.590	0.166	9.205	0.579
	p= 2	27.809	0.585	69.634	0.254	21.267	0.394
	p= 3	15.789	0.451	47.202	0.191	12.434	0.267
	p= 4	47.467	0.646	88.416	0.786	29.206	0.764
	p= 5	50.751	0.343	103.240	0.720	38.419	0.704

Note: 1) *, **, and *** implies rejection of the relevant null hypothesis at 10%, 5% and 1%significant level, respectively.

Therefore, according to the linearity tests, a liner VECM as in (4.3) explains the short-run dynamics of the data as well as the non-linear models. Consequently, any short-run deviation from the long-run sustainable path will be adjusted immediately, and the adjustment will be symmetric. Unlike the fiscal policy case, this result implies that the transactions costs associated with any adjustment to the long-run relation are not "large" enough to create disequilibria situation for which inaction is better due to the high cost associated with the adjustment. This different reaction to external disequilibria and fiscal disequilibria may be explained by the nature of the agents involved in the adjustment process. The adjustment to external disequilibria is done in a great proportion by private agents, while the adjustment to fiscal disequilibria is done by the government (executive and legislative branches). Since the private sector is able to adjust much quicker to new situation than the government, the result of a linear adjustment in the short-run external disequilibria should not be surprising.

The estimated VECMs with known cointegrating vector $[1,-1]$ are reported in Table 4.14. One surprising result is that the estimate for adjustment of MM_t to any short-run disequilibrium (γ_2) is not significantly different from zero for all the G-7 countries that present a sustainable current. This result implies that all the short-run adjustment to any disequilibrium is done via an adjustment in the exports[45]. And such adjustment presents the same magnitude and speed without taking in account the sign or magnitude of the disequilibria.

Table 4.14. Estimated VECM for X_t and MM_t with known cointegrating vector [1,-1]

Country		ΔX_t		ΔMM_t		Country		ΔX_t		ΔMM_t	
Canada	$X_{t-1} - MM_{t-1}$	0.0013	**	0.0008		Japan	$X_{t-1} - MM_{t-1}$	0.006	***	-0.0002	
(1976:1 - 1995:2)		(0.0006)		(0.0006)		(1957:1 - 1980:2)		(0.0004)		0.0004	
(p=2)	ΔX_{t-1}	-0.0821	*	0.1330	***	*(p=2)*	ΔX_{t-1}	0.0312		-0.2227	**
		(0.0487)		(0.0081)				(0.0851)		0.0946	
	ΔMM_{t-1}	0.0746	***	-0.0301	*		ΔMM_{t-1}	0.2717	***	0.5738	***
		(0.0094)		(0.0159)				(0.0711)		0.0791	
		T= 178						*T= 177*			
France	$X_{t-1} - MM_{t-1}$	0.0009	*	0.0006		United Kingdom	$X_{t-1} - MM_{t-1}$	0.002	***	0.0003	
(1970:1 - 2001:2)		(0.0005)		(0.0006)		(1957:1 - 1998:1)		(0.0006)		0.0009	
(p=2)	ΔX_{t-1}	-0.1952	*	-0.0221		*(p=2)*	ΔX_{t-1}	-0.3366	***	-0.1284	***
		(0.1066)		(0.1343)				(0.0888)		0.01266	
	ΔMM_{t-1}	0.3446	***	0.3469	***		ΔMM_{t-1}	0.3057	***	0.1094	
		(0.0844)		(0.1063)				(0.0660)		0.0941	
		T= 146						*T= 178*			
Italy	$X_{t-1} - MM_{t-1}$	0.001	*	0.0008		U.S.	$X_{t-1} - MM_{t-1}$	0.002	***	0.0004	
(1960:1 - 2001:1)		(0.0006)		(0.0007)		(1946:1 - 2001:2)		(0.0002)		(0.0003)	
(p=2)	ΔX_{t-1}	-0.0878		-0.2857	**	*(p=2)*	ΔX_{t-1}	0.0564		-0.1389	***
		(0.0945)		(0.1112)				(0.0910)		(0.0115)	
	ΔMM_{t-1}	0.1756	**	0.3774	***		ΔMM_{t-1}	0.0507	***	0.0954	***
		(0.0776)		(0.0913)				(0.0072)		(0.0091)	
		T= 163						*T= 178*			

Note: 1) Standard errors in parenthesis

2) p, the lag length was chosen by minimizing the multivariate version of the AIC.

3) *, **, and *** implies a significantly different than zero coefficient at 10%, 5% and 1%significant level, respectively.

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[45] This result is very interesting and more analysis should be done to explain this result. However, the required research is outside the scope of this dissertation.

In this subsection we found evidence in favor of the sustainability of the current account for Canada, France, Italy, Japan, the U.K., and the U.S.. Furthermore, we found a linear adjustment towards the long-run sustainable path for any short-run deviation, giving evidence to reject the idea that some adjustment costs are present so that the adjustment could not begin instantly.

5 Final Remarks and Conclusions

In this paper we have investigated the sustainability of the fiscal policies and current account of the G7 countries. An unsustainable situation was defined as a state of affairs when a change in the behavior of the economic agents is needed to bring back the system to a long-run sustainable path We make distinction between an unsustainable situation and a crises, since the later one is a situation for with the system has collapse after a long way along an unsustainable path

The theoretical framework used here, do not permit us to deal with crises, or to suggest what is the best timing of the adjustment needed if the system is in an unsustainable situation. However, the approach taken here has provided a step forward in the sustainability literature, since it allowed us to identify "unsustainable episodes", which was not possible in previous analysis.

In Chapter 2, we reviewed the relevant literature on the sustainable of a fiscal policy and a current account. In that chapter we identify two main approaches to the problem: i) a univariate and ii) a bivariate approach. The univariate approach exploits the univariate properties of the stock of public and the foreign debt. Meanwhile, the bivariate approach takes advantage of the time series properties of the flows that causes the stock of debt, e.g. in the case of de public debt those flows are the total revenues and the total expenditures (including the debt service).

The bivariate approach suggests a relatively straight forward test for sustainability. In the case of a sustainable fiscal policy, sustainability implies that total revenues and expenditures (measured as a portion of the GDP) need to be cointegrated with cointegrating vector [1,-1]. A similar test is provided for the sustainability of a current account. Researchers have been using traditional linear cointegration tools to tests for the sustainability of a fiscal policy or a current account. However, the idea of admitting the possibility of a change in regime in which the government budget had been expected to be balanced in present-value terms up until some date *t* and only after that date a permanent deficit was introduced or vice versa has been in mind of researchers. Different researchers have used different methods to capture this idea, for example using traditional cointegration tests for different sample sizes, or testing the stability of the estimated cointegrating vector throw out the whole study period. The intuition behind this idea is that there are large political and transaction costs to adjust the fiscal policy or the current account back to the long run relationship whenever a short disequilibria is present. Thus a "threshold" may exist, so that when ever the imbalance is "large" enough then the costs of the adjustment are lower than the benefits from it and thus the adjustment is made.

In Chapter 3 we discuss the concept of threshold cointegration, which perfectly captures the idea of a long-run relationship turning "on" and "off" according to the state of the system. For instance, if the deficit –fiscal or external– is "large" enough, then the system tends back to the long-run relationship. If the deficit is "small" enough, then no adjustment is done.

Chapter 3 reviews the cointegration tests available to test cointegration and threshold cointegration. We consider popular tests as Engel and Granger and Johansen cointegration tests that estimates the cointegrating vector as well as tests that use a known cointegrating vector, such as Horvath and Watson (1995) and the ADF test. We also consider the Enders and Granger (2000) and Berben and van Dijk (1999) tests that permit a two-regime TAR cointegrating error. With the help of a Monte Carlo study, we found that the effect of a misspecified delay parameter (*d*) varies according to the statistic. For example, the power of the ADF statistic decreases as the true *d* increases for both the case of a symmetric and asymmetric Band specification and symmetric and asymmetric EQ specification. Another statistic that presents a decrease in its power as *d* increases is the HW. On the other hand, the power of the Engel and Granger test is fairly stable, not matter the delay parameter or the specification, the power of this tests is consistently between 50% and 60%. The BVD and the Johansen statistics also present a fairly stable behavior with respect to changes in the delay parameter.

We also found a relatively low power of all the considered tests for particular combinations of parameters, so that there is not a single cointegration tests from ones considered that outperforms the others. Thus when it is suspected that a true cointegrating error's data generating process may be a three-regime TAR, we suggest that in practice researchers should use a set of cointegration tests and base the decision on the consensus of the different tests. In Chapter 3, the estimation of two and three-regime TVECM is discussed and two specification tests that may be used as linearity tests and/or specification tests.

After reviewing the econometric tools available, we apply, in Chapter 4, those tools to the sustainability problem for the G-7 countries. We found evidence in favor of the unsustainability of the Italian fiscal policy. For Canada, France, Japan[46], the U.K., and the U.S. we found evidence in favor of the sustainability of their fiscal policies for the period of study. Furthermore, we found evidence in favor of an EQ-TAR cointegrating error process. This permitted us to estimated thresholds for the fiscal deficit, so that "unsustainable episodes" where detected.

This non-linear behavior provides evidence in favor of the idea of a costly adjustment process that cannot be done instantly, since it requires political bargaining in different levels of the government. Thus, as the estimated model suggests, it is not expect an instant adjustment process to revert the system to the long-run relationship. But once the deviation achieves a "threshold", the situation is unmanageable and the authority will incur the cost needed to reach an adjustment.

On the other hand, we found that the German current account is not sustainable and the other 6 countries[47] present a sustainable current account. For the countries with sustainable current accounts no evidence in favor of a non-linear behavior was found. Thus a linear VECM was estimated.

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[46] Japan's period of study for the sustainability of the fiscal policy ends in 1980.

[47] Canada, France, Italy, Japan, the U.K., and the U.S.

An interesting founding, which needs to be studied in more detail, is that deviations from the long-run relationship are adjusted only by changes in the exports. This result seems surprising, but is consistent across countries.

Finally it is important to mention that the analysis in Chapter 3 points out the need of cointegration tests that present "good" power against the alternative hypothesis of a three-regime TAR. There is not a single cointegration tests designed for this purpose, and although previous Monte Carlo studies suggested that available tests perform well, we have found that that is not the case when a wider Monte Carlo study is considered. This result should encourage new research in this direction.

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Appendix 1. Results of the Monte Carlo Study.

In this Appendix we report the results of the Monte Carlo Study described in Chapter 3. Table A1, A2, A3, and A4 report the empirical rejection frequency of 5% cointegration tests when the true model has Symmetric Band specification, Asymmetric Band specification, Symmetric EQ specification, and Asymmetric EQ specification, respectively. Tables A5, A6, and A7 present the two tests with the larger power for small samples –*T*=100 –, moderate samples –*T*=250 –, and large samples –*T*=500–, respectively.

Table A1. Test for no-cointegration, empirical rejection frequency of 5% tests
Symmetric Band-Specification (ϕ=*-0.1*, *d*=*1*)

c	*T*	ADF	Engel-Granger	Enders-Granger	Johansen		HW	BVD
					Trace	Max		
3	100	0.199	0.542	0.071	0.180	0.100	0.320	0.174
3	250	0.777	0.609	0.144	0.264	0.165	0.618	0.349
3	500	0.998	0.652	0.565	0.597	0.427	0.986	0.872
5	100	0.095	0.533	0.063	0.155	0.086	0.275	0.161
5	250	0.298	0.556	0.105	0.229	0.134	0.372	0.263
5	500	0.928	0.569	0.228	0.322	0.184	0.727	0.618
10	100	0.076	0.525	0.049	0.148	0.079	0.195	0.136
10	250	0.077	0.522	0.083	0.159	0.096	0.315	0.225
10	500	0.155	0.560	0.138	0.207	0.131	0.369	0.336

Table A1. (Cont.) Test for no-cointegration, empirical rejection frequency of 5% tests
Symmetric Band-Specification (ϕ=*-0.1*, *d*=*3*)

c	*T*	ADF	Engel-Granger	Enders-Granger	Johansen		HW	BVD
					Trace	Max		
3	100	0.148	0.519	0.061	0.183	0.112	0.305	0.165
3	250	0.722	0.555	0.124	0.267	0.142	0.543	0.353
3	500	0.996	0.627	0.470	0.565	0.412	0.981	0.817
5	100	0.094	0.521	0.051	0.164	0.098	0.300	0.153
5	250	0.234	0.518	0.111	0.219	0.138	0.388	0.271
5	500	0.890	0.552	0.183	0.308	0.178	0.677	0.573
10	100	0.065	0.507	0.064	0.166	0.098	0.238	0.163
10	250	0.067	0.542	0.082	0.170	0.098	0.297	0.213
10	500	0.116	0.532	0.107	0.202	0.118	0.354	0.347

Table A1. (Cont.) Test for no-cointegration, empirical rejection frequency of 5% tests
Symmetric Band- Specification (ϕ=*-0.1, d=6*)

c	*T*	ADF	Engel-Granger	Enders-Granger	Johansen		HW	BVD
					Trace	Max		
3	100	0.130	0.529	0.075	0.188	0.101	0.356	0.188
3	250	0.664	0.570	0.146	0.311	0.191	0.527	0.311
3	500	1.000	0.602	0.388	0.547	0.380	0.961	0.739
5	100	0.066	0.516	0.067	0.173	0.099	0.292	0.168
5	250	0.186	0.541	0.123	0.242	0.148	0.442	0.295
5	500	0.819	0.581	0.212	0.333	0.209	0.619	0.528
10	100	0.063	0.510	0.049	0.159	0.086	0.242	0.155
10	250	0.058	0.517	0.112	0.200	0.126	0.339	0.241
10	500	0.086	0.520	0.136	0.241	0.148	0.384	0.381

Table A1. (Cont.) Test for no-cointegration, empirical rejection frequency of 5% tests
Symmetric Band- Specification (ϕ=*-0.1, d=9*)

c	*T*	ADF	Engel-Granger	Enders-Granger	Johansen		HW	BVD
					Trace	Max		
3	100	0.128	0.580	0.093	0.202	0.115	0.363	0.198
3	250	0.578	0.550	0.175	0.304	0.208	0.561	0.316
3	500	0.993	0.597	0.362	0.555	0.364	0.959	0.699
5	100	0.066	0.515	0.070	0.165	0.093	0.309	0.167
5	250	0.148	0.544	0.172	0.252	0.172	0.436	0.297
5	500	0.776	0.575	0.220	0.332	0.227	0.606	0.509
10	100	0.059	0.559	0.075	0.172	0.101	0.233	0.166
10	250	0.048	0.532	0.109	0.198	0.136	0.326	0.243
10	500	0.088	0.538	0.158	0.243	0.163	0.375	0.371

Table A1. (Cont.) Test for no-cointegration, empirical rejection frequency of 5% tests
Symmetric Band- Specification (ϕ=*-0.1, d=12*)

c	*T*	ADF	Engel-Granger	Enders-Granger	Johansen		HW	BVD
					Trace	Max		
3	100	0.099	0.551	0.086	0.211	0.114	0.364	0.191
3	250	0.564	0.568	0.197	0.312	0.216	0.558	0.349
3	500	0.994	0.594	0.350	0.558	0.420	0.929	0.640
5	100	0.063	0.511	0.090	0.193	0.110	0.318	0.209
5	250	0.120	0.538	0.195	0.287	0.178	0.489	0.313
5	500	0.728	0.564	0.246	0.388	0.293	0.627	0.494
10	100	0.055	0.523	0.079	0.160	0.093	0.233	0.201
10	250	0.040	0.501	0.152	0.253	0.158	0.386	0.275
10	500	0.059	0.529	0.242	0.309	0.229	0.435	0.411

Table A1. (Cont.) Test for no-cointegration, empirical rejection frequency of 5% tests Symmetric Band-Specification ($\phi=-0.6$, $d=1$)

c	T	ADF	Engel-Granger	Enders-Granger	Johansen		HW	BVD
					Trace	Max		
3	100	0.828	0.570	0.087	0.341	0.210	0.680	0.305
3	250	0.998	0.676	0.607	0.869	0.806	1.000	0.911
3	500	1.000	0.781	0.997	0.797	0.797	1.000	1.000
5	100	0.255	0.577	0.073	0.227	0.143	0.389	0.246
5	250	0.895	0.611	0.176	0.376	0.242	0.765	0.598
5	500	1.000	0.663	0.635	0.810	0.695	1.000	0.982
10	100	0.057	0.518	0.061	0.194	0.120	0.267	0.183
10	250	0.088	0.532	0.098	0.222	0.126	0.354	0.358
10	500	0.436	0.564	0.151	0.272	0.162	0.447	0.604

Table A1. (Cont.) Test for no-cointegration, empirical rejection frequency of 5% tests Symmetric Band-Specification ($\phi=-0.6$, $d=3$)

c	T	ADF	Engel-Granger	Enders-Granger	Johansen		HW	BVD
					Trace	Max		
3	100	0.585	0.567	0.099	0.418	0.336	0.652	0.212
3	250	0.801	0.603	0.177	0.562	0.457	0.893	0.490
3	500	0.992	0.669	0.626	0.761	0.634	0.999	0.942
5	100	0.083	0.551	0.089	0.350	0.271	0.484	0.218
5	250	0.450	0.551	0.141	0.384	0.307	0.602	0.462
5	500	0.756	0.596	0.238	0.404	0.267	0.790	0.871
10	100	0.055	0.495	0.078	0.270	0.212	0.381	0.185
10	250	0.054	0.517	0.110	0.278	0.208	0.386	0.379
10	500	0.153	0.534	0.180	0.260	0.170	0.415	0.676

Table A1. (Cont.) Test for no-cointegration, empirical rejection frequency of 5% tests Symmetric Band-Specification ($\phi=-0.6$, $d=6$)

c	T	ADF	Engel-Granger	Enders-Granger	Johansen		HW	BVD
					Trace	Max		
3	100	0.628	0.525	0.293	0.742	0.697	0.845	0.432
3	250	0.701	0.549	0.412	0.790	0.735	0.926	0.506
3	500	0.547	0.563	0.311	0.849	0.780	1.000	0.559
5	100	0.110	0.515	0.328	0.709	0.653	0.811	0.477
5	250	0.444	0.536	0.409	0.667	0.639	0.761	0.560
5	500	0.367	0.529	0.396	0.590	0.548	0.793	0.640
10	100	0.034	0.535	0.253	0.585	0.535	0.663	0.379
10	250	0.037	0.526	0.383	0.565	0.517	0.654	0.545
10	500	0.059	0.538	0.387	0.516	0.461	0.628	0.740

Table A1. (Cont.) Test for no-cointegration, empirical rejection frequency of 5% tests Symmetric Band-Specification ($\phi=-0.6$, $d=9$)

c	T	ADF	Engel-Granger	Enders-Granger	Johansen		HW	BVD
					Trace	Max		
3	100	0.663	0.517	0.448	0.828	0.785	0.935	0.592
3	250	0.824	0.512	0.641	0.902	0.868	0.977	0.730
3	500	0.893	0.589	0.606	0.933	0.910	1.000	0.785
5	100	0.106	0.500	0.453	0.797	0.735	0.893	0.600
5	250	0.613	0.497	0.701	0.840	0.820	0.876	0.795
5	500	0.726	0.547	0.654	0.778	0.749	0.868	0.823
10	100	0.017	0.515	0.424	0.727	0.678	0.832	0.557
10	250	0.019	0.509	0.615	0.741	0.710	0.804	0.705
10	500	0.088	0.519	0.673	0.726	0.698	0.789	0.864

Table A1. (Cont.) Test for no-cointegration, empirical rejection frequency of 5% tests Symmetric Band-Specification ($\phi=-0.6$, $d=12$)

c	T	ADF	Engel-Granger	Enders-Granger	Johansen		HW	BVD
					Trace	Max		
3	100	0.704	0.524	0.492	0.833	0.752	0.967	0.642
3	250	0.910	0.552	0.848	0.954	0.938	0.984	0.892
3	500	0.977	0.551	0.839	0.971	0.957	0.999	0.930
5	100	0.114	0.521	0.408	0.800	0.717	0.938	0.594
5	250	0.692	0.510	0.825	0.908	0.889	0.942	0.869
5	500	0.825	0.536	0.823	0.882	0.866	0.937	0.898
10	100	0.010	0.517	0.391	0.788	0.714	0.897	0.573
10	250	0.014	0.511	0.777	0.853	0.837	0.894	0.834
10	500	0.118	0.487	0.827	0.855	0.833	0.893	0.932

Table A1. (Cont.) Test for no-cointegration, empirical rejection frequency of 5% tests Symmetric Band-Specification ($\phi=-0.9$, $d=1$)

c	T	ADF	Engel-Granger	Enders-Granger	Johansen		HW	BVD
					Trace	Max		
3	100	0.932	0.617	0.150	0.403	0.249	0.836	0.409
3	250	1.000	0.705	0.782	0.949	0.934	1.000	0.971
3	500	1.000	0.794	1.000	1.000	1.000	1.000	1.000
5	100	0.339	0.562	0.080	0.219	0.135	0.425	0.264
5	250	0.952	0.590	0.243	0.433	0.286	0.872	0.742
5	500	0.999	0.665	0.767	0.908	0.858	1.000	0.994
10	100	0.054	0.509	0.065	0.179	0.119	0.289	0.198
10	250	0.099	0.529	0.093	0.213	0.146	0.342	0.395
10	500	0.555	0.564	0.143	0.260	0.157	0.465	0.664

Table A1. (Cont.) Test for no-cointegration, empirical rejection frequency of 5% tests
Symmetric Band-Specification ($\phi=-0.9, d=3$)

c	*T*	ADF	Engel-Granger	Enders-Granger	Johansen		HW	BVD
					Trace	Max		
3	100	0.487	0.544	0.167	0.608	0.548	0.788	0.304
3	250	0.307	0.539	0.163	0.497	0.410	0.868	0.336
3	500	0.879	0.616	0.222	0.583	0.468	1.000	0.640
5	100	0.232	0.518	0.176	0.545	0.492	0.682	0.294
5	250	0.111	0.553	0.199	0.373	0.316	0.561	0.396
5	500	0.192	0.556	0.192	0.230	0.159	0.697	0.586
10	100	0.058	0.537	0.126	0.422	0.365	0.520	0.246
10	250	0.036	0.498	0.149	0.316	0.254	0.468	0.403
10	500	0.055	0.503	0.224	0.193	0.145	0.426	0.766

Table A1. (Cont.) Test for no-cointegration, empirical rejection frequency of 5% tests
Symmetric Specification TVECM ($\phi=-0.9, d=6$)

c	*T*	ADF	Engel-Granger	Enders-Granger	Johansen		HW	BVD
					Trace	Max		
3	100	0.381	0.526	0.320	0.966	0.960	0.979	0.469
3	250	0.596	0.571	0.384	0.968	0.958	0.992	0.684
3	500	0.920	0.604	0.631	0.994	0.986	1.000	0.888
5	100	0.440	0.519	0.634	0.899	0.883	0.942	0.755
5	250	0.570	0.510	0.749	0.882	0.863	0.918	0.827
5	500	0.485	0.519	0.697	0.896	0.861	0.959	0.819
10	100	0.028	0.532	0.627	0.839	0.822	0.876	0.737
10	250	0.147	0.491	0.734	0.840	0.810	0.879	0.832
10	500	0.144	0.527	0.728	0.807	0.787	0.858	0.898

Table A1. (Cont.) Test for no-cointegration, empirical rejection frequency of 5% tests
Symmetric Band-Specification ($\phi=-0.9, d=9$)

c	*T*	ADF	Engel-Granger	Enders-Granger	Johansen		HW	BVD
					Trace	Max		
3	100	0.770	0.561	0.281	0.975	0.972	0.995	0.443
3	250	0.933	0.617	0.794	0.997	0.995	1.000	0.915
3	500	0.997	0.725	0.966	0.999	0.999	1.000	0.989
5	100	0.465	0.523	0.602	0.924	0.892	0.979	0.742
5	250	0.810	0.522	0.889	0.972	0.969	0.983	0.944
5	500	0.907	0.562	0.916	0.974	0.966	0.994	0.957
10	100	0.020	0.509	0.649	0.877	0.846	0.960	0.801
10	250	0.088	0.505	0.895	0.927	0.922	0.949	0.937
10	500	0.437	0.496	0.914	0.924	0.920	0.950	0.966

Table A1. (Cont.) Test for no-cointegration, empirical rejection frequency of 5% tests Symmetric Band-Specification ($\phi=-0.9$, $d=12$)

c	T	ADF	Engel-Granger	Enders-Granger	Johansen Trace	Johansen Max	HW	BVD
3	100	0.918	0.529	0.950	0.994	0.987	1.000	0.985
3	250	0.984	0.549	0.995	1.000	1.000	1.000	0.999
3	500	1.000	0.641	1.000	1.000	1.000	1.000	1.000
5	100	0.605	0.510	0.678	0.912	0.878	0.987	0.844
5	250	0.873	0.516	0.976	0.987	0.988	0.992	0.985
5	500	0.941	0.549	0.987	0.989	0.988	0.998	0.994
10	100	0.022	0.531	0.565	0.861	0.811	0.976	0.757
10	250	0.084	0.502	0.966	0.976	0.972	0.985	0.974
10	500	0.485	0.490	0.972	0.983	0.980	0.983	0.986

Table A2. Test for no-cointegration, empirical rejection frequency of 5% tests Asymmetric BAND-Specification ($\phi=-0.1$, $d=1$)

$c^{(1)}$	$c^{(2)}$	T	ADF	Engel-Granger	Enders-Granger	Johansen Trace	Johansen Max	HW	BVD
-5	3	100	0.149	0.591	0.061	0.157	0.101	0.284	0.178
-5	3	250	0.469	0.644	0.131	0.221	0.132	0.432	0.319
-5	3	500	0.975	0.698	0.363	0.448	0.272	0.896	0.746
-3	5	100	0.125	0.500	0.050	0.163	0.105	0.276	0.156
-3	5	250	0.466	0.484	0.116	0.236	0.130	0.443	0.318
-3	5	500	0.983	0.478	0.337	0.433	0.267	0.904	0.721
-10	5	100	0.074	0.581	0.060	0.161	0.098	0.248	0.150
-10	5	250	0.145	0.615	0.082	0.179	0.113	0.300	0.214
-10	5	500	0.383	0.700	0.131	0.229	0.129	0.414	0.406
-3	10	100	0.101	0.385	0.041	0.158	0.098	0.244	0.140
-3	10	250	0.214	0.369	0.094	0.193	0.114	0.332	0.227
-3	10	500	0.372	0.309	0.148	0.253	0.145	0.501	0.467

Table A2.(Cont.) Test for no-cointegration, empirical rejection frequency of 5% tests Asymmetric BAND -Specification ($\phi=-0.1$, $d=3$)

$c^{(1)}$	$c^{(2)}$	T	ADF	Engel-Granger	Enders-Granger	Johansen		HW	BVD
						Trace	Max		
-5	3	100	0.128	0.575	0.056	0.171	0.098	0.306	0.167
-5	3	250	0.444	0.626	0.125	0.225	0.130	0.435	0.316
-5	3	500	0.960	0.682	0.292	0.437	0.253	0.864	0.715
-3	5	100	0.132	0.518	0.072	0.176	0.108	0.310	0.160
-3	5	250	0.412	0.483	0.118	0.241	0.153	0.446	0.291
-3	5	500	0.958	0.477	0.297	0.400	0.223	0.879	0.698
-10	5	100	0.073	0.601	0.051	0.149	0.097	0.237	0.140
-10	5	250	0.126	0.632	0.083	0.173	0.114	0.329	0.229
-10	5	500	0.376	0.677	0.141	0.227	0.134	0.412	0.407
-3	10	100	0.109	0.446	0.063	0.170	0.114	0.246	0.147
-3	10	250	0.205	0.407	0.090	0.174	0.103	0.310	0.230
-3	10	500	0.378	0.393	0.148	0.250	0.154	0.463	0.456

Table A2.(Cont.) Test for no-cointegration, empirical rejection frequency of 5% tests Asymmetric BAND - Specification ($\phi=-0.1$, $d=6$)

$c^{(1)}$	$c^{(2)}$	T	ADF	Engel-Granger	Enders-Granger	Johansen		HW	BVD
						Trace	Max		
-5	3	100	0.089	0.595	0.092	0.207	0.112	0.325	0.186
-5	3	250	0.377	0.617	0.126	0.255	0.156	0.446	0.307
-5	3	500	0.927	0.670	0.247	0.421	0.280	0.803	0.609
-3	5	100	0.100	0.475	0.067	0.200	0.116	0.323	0.159
-3	5	250	0.375	0.494	0.142	0.245	0.166	0.442	0.291
-3	5	500	0.939	0.503	0.239	0.408	0.247	0.816	0.599
-10	5	100	0.068	0.600	0.071	0.174	0.105	0.253	0.178
-10	5	250	0.097	0.602	0.120	0.223	0.147	0.356	0.249
-10	5	500	0.322	0.654	0.177	0.278	0.177	0.429	0.419
-3	10	100	0.088	0.404	0.049	0.172	0.112	0.258	0.154
-3	10	250	0.212	0.411	0.109	0.213	0.131	0.359	0.229
-3	10	500	0.356	0.365	0.180	0.269	0.175	0.468	0.455

Table A2.(Cont.) Test for no-cointegration, empirical rejection frequency of 5% tests Asymmetric BAND - Specification (ϕ=*-0.1, d=9*)

$c^{(1)}$	$c^{(2)}$	*T*	ADF	Engel-Granger	Enders-Granger	Johansen		HW	BVD
						Trace	Max		
-5	3	100	0.084	0.594	0.102	0.212	0.120	0.339	0.212
-5	3	250	0.321	0.597	0.165	0.274	0.182	0.470	0.323
-5	3	500	0.908	0.670	0.234	0.389	0.261	0.766	0.561
-3	5	100	0.099	0.479	0.074	0.188	0.098	0.323	0.190
-3	5	250	0.330	0.521	0.161	0.266	0.190	0.462	0.310
-3	5	500	0.916	0.482	0.255	0.406	0.268	0.787	0.577
-10	5	100	0.061	0.585	0.063	0.176	0.105	0.261	0.173
-10	5	250	0.089	0.593	0.139	0.252	0.170	0.398	0.264
-10	5	500	0.318	0.631	0.187	0.294	0.207	0.434	0.385
-3	10	100	0.076	0.450	0.081	0.188	0.106	0.289	0.208
-3	10	250	0.221	0.435	0.165	0.269	0.186	0.440	0.310
-3	10	500	0.346	0.363	0.207	0.304	0.204	0.491	0.431

Table A2.(Cont.) Test for no-cointegration, empirical rejection frequency of 5% tests Asymmetric BAND - Specification (ϕ=*-0.1, d=12*)

$c^{(1)}$	$c^{(2)}$	*T*	ADF	Engel-Granger	Enders-Granger	Johansen		HW	BVD
						Trace	Max		
-5	3	100	0.088	0.598	0.071	0.162	0.085	0.338	0.173
-5	3	250	0.297	0.605	0.182	0.301	0.195	0.491	0.315
-5	3	500	0.890	0.646	0.276	0.448	0.345	0.772	0.544
-3	5	100	0.082	0.485	0.077	0.180	0.104	0.340	0.182
-3	5	250	0.277	0.501	0.196	0.301	0.198	0.505	0.329
-3	5	500	0.878	0.500	0.298	0.427	0.329	0.768	0.538
-10	5	100	0.065	0.581	0.074	0.156	0.095	0.276	0.170
-10	5	250	0.089	0.603	0.165	0.267	0.182	0.407	0.290
-10	5	500	0.290	0.632	0.253	0.345	0.249	0.491	0.415
-3	10	100	0.085	0.443	0.083	0.166	0.101	0.281	0.186
-3	10	250	0.205	0.440	0.193	0.277	0.184	0.457	0.312
-3	10	500	0.331	0.377	0.256	0.336	0.259	0.489	0.435

Table A2.(Cont.) Test for no-cointegration, empirical rejection frequency of 5% tests Asymmetric BAND -Specification ($\phi=-0.6$, $d=1$)

$c^{(1)}$	$c^{(2)}$	T	ADF	Engel-Granger	Enders-Granger	Johansen Trace	Johansen Max	HW	BVD
-5	3	100	0.450	0.642	0.083	0.236	0.136	0.456	0.277
-5	3	250	0.966	0.736	0.320	0.536	0.380	0.968	0.761
-5	3	500	1.000	0.819	0.921	0.984	0.975	1.000	0.998
-3	5	100	0.440	0.500	0.076	0.259	0.159	0.467	0.253
-3	5	250	0.948	0.473	0.309	0.556	0.381	0.967	0.768
-3	5	500	1.000	0.494	0.889	0.969	0.956	1.000	0.998
-10	5	100	0.125	0.637	0.063	0.187	0.114	0.315	0.198
-10	5	250	0.320	0.670	0.117	0.257	0.175	0.416	0.425
-10	5	500	0.748	0.745	0.192	0.346	0.219	0.743	0.811
-3	10	100	0.388	0.083	0.205	0.126	0.345	0.211	0.279
-3	10	250	0.279	0.348	0.121	0.249	0.149	0.469	0.471
-3	10	500	0.456	0.475	0.706	0.766	0.734	0.820	0.850

Table A2.(Cont.) Test for no-cointegration, empirical rejection frequency of 5% tests Asymmetric BAND -Specification ($\phi=-0.6$, $d=3$)

$c^{(1)}$	$c^{(2)}$	T	ADF	Engel-Granger	Enders-Granger	Johansen Trace	Johansen Max	HW	BVD
-5	3	100	0.325	0.597	0.102	0.379	0.309	0.560	0.226
-5	3	250	0.588	0.630	0.163	0.449	0.357	0.755	0.486
-5	3	500	0.878	0.717	0.352	0.505	0.387	0.957	0.917
-3	5	100	0.291	0.507	0.099	0.401	0.310	0.561	0.226
-3	5	250	0.573	0.482	0.152	0.435	0.313	0.775	0.486
-3	5	500	0.903	0.510	0.334	0.532	0.408	0.966	0.927
-10	5	100	0.066	0.598	0.076	0.297	0.215	0.421	0.199
-10	5	250	0.233	0.620	0.132	0.340	0.265	0.488	0.445
-10	5	500	0.348	0.689	0.182	0.294	0.206	0.518	0.810
-3	10	100	0.234	0.390	0.092	0.328	0.253	0.458	0.217
-3	10	250	0.253	0.381	0.162	0.336	0.250	0.478	0.454
-3	10	500	0.367	0.345	0.236	0.346	0.225	0.669	0.872

Table A2.(Cont.) Test for no-cointegration, empirical rejection frequency of 5% tests Asymmetric BAND -Specification (*ϕ=-0.6, d=6*)

$c^{(1)}$	$c^{(2)}$	*T*	ADF	Engel-Granger	Enders-Granger	Johansen		HW	BVD
						Trace	Max		
-5	3	100	0.337	0.561	0.310	0.706	0.655	0.817	0.449
-5	3	250	0.613	0.577	0.464	0.716	0.676	0.849	0.574
-5	3	500	0.451	0.590	0.373	0.725	0.641	0.944	0.616
-3	5	100	0.343	0.485	0.304	0.689	0.650	0.810	0.455
-3	5	250	0.555	0.484	0.430	0.691	0.650	0.813	0.516
-3	5	500	0.423	0.487	0.350	0.686	0.586	0.953	0.605
-10	5	100	0.049	0.567	0.267	0.616	0.564	0.710	0.401
-10	5	250	0.223	0.551	0.382	0.600	0.563	0.690	0.561
-10	5	500	0.209	0.599	0.424	0.528	0.484	0.660	0.738
-3	10	100	0.270	0.446	0.294	0.642	0.612	0.741	0.407
-3	10	250	0.317	0.421	0.401	0.640	0.605	0.732	0.567
-3	10	500	0.261	0.415	0.401	0.566	0.524	0.664	0.703

Table A2.(Cont.) Test for no-cointegration, empirical rejection frequency of 5% tests Asymmetric BAND -Specification (*ϕ=-0.6, d=9*)

$c^{(1)}$	$c^{(2)}$	*T*	ADF	Engel-Granger	Enders-Granger	Johansen		HW	BVD
						Trace	Max		
-5	3	100	0.391	0.551	0.480	0.800	0.743	0.913	0.627
-5	3	250	0.754	0.550	0.678	0.843	0.816	0.904	0.751
-5	3	500	0.810	0.551	0.654	0.834	0.786	0.963	0.831
-3	5	100	0.407	0.494	0.441	0.780	0.736	0.886	0.582
-3	5	250	0.739	0.516	0.703	0.836	0.811	0.900	0.771
-3	5	500	0.824	0.505	0.642	0.842	0.800	0.971	0.805
-10	5	100	0.062	0.536	0.419	0.727	0.668	0.836	0.571
-10	5	250	0.350	0.507	0.685	0.787	0.773	0.841	0.774
-10	5	500	0.390	0.542	0.697	0.743	0.714	0.810	0.869
-3	10	100	0.386	0.494	0.448	0.770	0.713	0.873	0.587
-3	10	250	0.457	0.513	0.712	0.819	0.797	0.873	0.800
-3	10	500	0.440	0.456	0.698	0.786	0.760	0.835	0.854

Table A2.(Cont.) Test for no-cointegration, empirical rejection frequency of 5% tests Asymmetric BAND -Specification (ϕ=-0.6, d=12)

$c^{(1)}$	$c^{(2)}$	T	ADF	Engel-Granger	Enders-Granger	Johansen		HW	BVD
						Trace	Max		
-5	3	100	0.453	0.539	0.437	0.806	0.745	0.953	0.617
-5	3	250	0.801	0.539	0.868	0.933	0.919	0.956	0.899
-5	3	500	0.889	0.542	0.816	0.937	0.914	0.988	0.905
-3	5	100	0.446	0.535	0.443	0.818	0.771	0.944	0.613
-3	5	250	0.791	0.491	0.844	0.928	0.915	0.957	0.881
-3	5	500	0.884	0.513	0.802	0.931	0.903	0.981	0.905
-10	5	100	0.072	0.510	0.374	0.759	0.679	0.905	0.552
-10	5	250	0.417	0.439	0.798	0.891	0.874	0.930	0.851
-10	5	500	0.510	0.482	0.832	0.864	0.855	0.888	0.919
-3	10	100	0.392	0.552	0.389	0.779	0.718	0.928	0.580
-3	10	250	0.531	0.573	0.812	0.901	0.882	0.922	0.865
-3	10	500	0.607	0.589	0.843	0.878	0.865	0.911	0.931

Table A2.(Cont.) Test for no-cointegration, empirical rejection frequency of 5% tests Asymmetric EQ-Specification (ϕ=-0.9, d=1)

$c^{(1)}$	$c^{(2)}$	T	ADF	Engel-Granger	Enders-Granger	Johansen		HW	BVD
						Trace	Max		
-5	3	100	0.578	0.681	0.096	0.277	0.159	0.537	0.331
-5	3	250	0.981	0.769	0.419	0.699	0.597	0.987	0.884
-5	3	500	1.000	0.860	0.966	0.987	0.988	1.000	1.000
-3	5	100	0.602	0.475	0.099	0.279	0.175	0.572	0.330
-3	5	250	0.971	0.474	0.447	0.707	0.566	0.988	0.873
-3	5	500	1.000	0.490	0.980	0.990	0.985	1.000	1.000
-10	5	100	0.145	0.608	0.060	0.207	0.134	0.319	0.219
-10	5	250	0.378	0.706	0.136	0.283	0.189	0.463	0.531
-10	5	500	0.792	0.763	0.260	0.413	0.285	0.834	0.904
-3	10	100	0.227	0.385	0.078	0.223	0.142	0.342	0.261
-3	10	250	0.321	0.317	0.161	0.337	0.208	0.556	0.571
-3	10	500	0.198	0.359	0.228	0.228	0.155	0.486	0.698

Table A2.(Cont.) Test for no-cointegration, empirical rejection frequency of 5% tests Asymmetric BAND -Specification (*ϕ=-0.9, d=3*)

$c^{(1)}$	$c^{(2)}$	*T*	ADF	Engel-Granger	Enders-Granger	Johansen		HW	BVD
						Trace	Max		
-5	3	100	0.353	0.597	0.185	0.598	0.532	0.715	0.294
-5	3	250	0.155	0.605	0.180	0.378	0.333	0.629	0.330
-5	3	500	0.426	0.671	0.214	0.311	0.225	0.958	0.582
-3	5	100	0.344	0.503	0.168	0.570	0.523	0.693	0.281
-3	5	250	0.144	0.493	0.204	0.398	0.333	0.655	0.356
-3	5	500	0.431	0.516	0.213	0.302	0.215	0.966	0.598
-10	5	100	0.127	0.595	0.140	0.486	0.426	0.589	0.275
-10	5	250	0.084	0.627	0.163	0.317	0.262	0.498	0.424
-10	5	500	0.144	0.647	0.269	0.245	0.195	0.482	0.751
-3	10	100	0.218	0.433	0.129	0.486	0.421	0.620	0.264
-3	10	250	0.138	0.404	0.170	0.351	0.288	0.530	0.415
-3	10	500	0.186	0.344	0.254	0.244	0.169	0.515	0.710

Table A2.(Cont.) Test for no-cointegration, empirical rejection frequency of 5% tests Asymmetric BAND-Specification (*ϕ=-0.9, d=6*)

$c^{(1)}$	$c^{(2)}$	*T*	ADF	Engel-Granger	Enders-Granger	Johansen		HW	BVD
						Trace	Max		
-5	3	100	0.525	0.543	0.534	0.925	0.909	0.954	0.670
-5	3	250	0.607	0.552	0.637	0.920	0.907	0.956	0.802
-5	3	500	0.629	0.581	0.626	0.934	0.907	0.995	0.824
-3	5	100	0.523	0.488	0.529	0.931	0.921	0.960	0.670
-3	5	250	0.586	0.495	0.621	0.918	0.907	0.961	0.792
-3	5	500	0.633	0.483	0.616	0.934	0.911	0.996	0.829
-10	5	100	0.277	0.492	0.621	0.858	0.834	0.907	0.745
-10	5	250	0.389	0.475	0.736	0.846	0.833	0.882	0.843
-10	5	500	0.313	0.494	0.752	0.851	0.827	0.864	0.876
-3	10	100	0.437	0.513	0.640	0.889	0.872	0.917	0.749
-3	10	250	0.527	0.550	0.736	0.859	0.837	0.895	0.823
-3	10	500	0.504	0.561	0.762	0.870	0.857	0.905	0.860

Table A2.(Cont.) Test for no-cointegration, empirical rejection frequency of 5% tests Asymmetric BAND -Specification (ϕ=-*0.9*, *d*=*9*)

$c^{(1)}$	$c^{(2)}$	T	ADF	Engel-Granger	Enders-Granger	Johansen		HW	BVD
						Trace	Max		
-5	3	100	0.648	0.568	0.446	0.945	0.923	0.987	0.605
-5	3	250	0.827	0.614	0.831	0.987	0.984	0.994	0.927
-5	3	500	0.943	0.656	0.933	0.992	0.988	1.000	0.966
-3	5	100	0.659	0.482	0.439	0.951	0.943	0.992	0.607
-3	5	250	0.825	0.474	0.845	0.985	0.979	0.993	0.939
-3	5	500	0.935	0.461	0.929	0.992	0.986	1.000	0.971
-10	5	100	0.291	0.421	0.676	0.895	0.851	0.961	0.807
-10	5	250	0.506	0.424	0.922	0.951	0.946	0.965	0.948
-10	5	500	0.732	0.380	0.930	0.954	0.946	0.962	0.960
-3	10	100	0.552	0.578	0.642	0.874	0.833	0.966	0.792
-3	10	250	0.646	0.634	0.926	0.952	0.948	0.971	0.951
-3	10	500	0.786	0.670	0.930	0.964	0.956	0.972	0.968

Table A2.(Cont.) Test for no-cointegration, empirical rejection frequency of 5% tests Asymmetric BAND -Specification (ϕ=-*0.9*, *d*=*12*)

$c^{(1)}$	$c^{(2)}$	T	ADF	Engel-Granger	Enders-Granger	Johansen		HW	BVD
						Trace	Max		
-5	3	100	0.853	0.579	0.879	0.964	0.957	0.998	0.936
-5	3	250	0.944	0.596	0.995	0.999	0.998	0.999	0.997
-5	3	500	0.982	0.637	0.991	0.999	0.998	0.999	0.995
-3	5	100	0.856	0.485	0.861	0.975	0.963	0.997	0.932
-3	5	250	0.958	0.475	0.994	0.998	0.998	1.000	0.997
-3	5	500	0.984	0.475	0.995	1.000	1.000	1.000	0.997
-10	5	100	0.291	0.438	0.563	0.854	0.803	0.989	0.773
-10	5	250	0.556	0.379	0.971	0.983	0.982	0.988	0.982
-10	5	500	0.766	0.377	0.977	0.988	0.984	0.990	0.993
-3	10	100	0.616	0.627	0.561	0.854	0.811	0.989	0.767
-3	10	250	0.695	0.667	0.967	0.981	0.980	0.986	0.976
-3	10	500	0.832	0.663	0.965	0.983	0.979	0.987	0.982

Table A3.Test for no-cointegration, empirical rejection frequency of 5% tests
Symmetric EQ-Specification (ϕ=-0.1, d=1)

c	*T*	ADF	Engel-Granger	Enders-Granger	Johansen		HW	BVD
					Trace	Max		
3	100	0.592	0.547	0.077	0.226	0.133	0.493	0.246
3	250	0.997	0.619	0.435	0.586	0.419	0.974	0.691
3	500	1.000	0.701	0.960	0.989	0.986	1.000	0.993
5	100	0.214	0.533	0.071	0.219	0.131	0.347	0.197
5	250	0.913	0.607	0.182	0.325	0.196	0.719	0.537
5	500	1.000	0.636	0.655	0.795	0.664	1.000	0.960
10	100	0.063	0.534	0.061	0.189	0.109	0.288	0.184
10	250	0.129	0.530	0.090	0.221	0.145	0.376	0.341
10	500	0.585	0.580	0.167	0.288	0.184	0.491	0.628

Table A3.(Cont.) Test for no-cointegration, empirical rejection frequency of 5% tests
Symmetric EQ-Specification (ϕ=-0.1, d=3)

c	*T*	ADF	Engel-Granger	Enders-Granger	Johansen		HW	BVD
					Trace	Max		
3	100	0.511	0.559	0.098	0.221	0.120	0.423	0.205
3	250	0.993	0.631	0.420	0.549	0.366	0.935	0.673
3	500	1.000	0.718	0.968	0.980	0.980	1.000	0.993
5	100	0.242	0.531	0.068	0.198	0.098	0.365	0.183
5	250	0.917	0.612	0.211	0.302	0.173	0.669	0.459
5	500	0.999	0.624	0.746	0.804	0.669	0.999	0.945
10	100	0.073	0.531	0.064	0.165	0.108	0.250	0.160
10	250	0.158	0.558	0.103	0.173	0.113	0.348	0.270
10	500	0.722	0.567	0.171	0.293	0.182	0.515	0.546

Table A3.(Cont.) Test for no-cointegration, empirical rejection frequency of 5% tests
Symmetric EQ- Specification (ϕ=-0.1, d=6)

c	*T*	ADF	Engel-Granger	Enders-Granger	Johansen		HW	BVD
					Trace	Max		
3	100	0.396	0.539	0.097	0.144	0.077	0.320	0.226
3	250	0.993	0.635	0.446	0.387	0.201	0.844	0.654
3	500	1.000	0.702	0.986	0.919	0.901	1.000	0.998
5	100	0.203	0.564	0.084	0.141	0.081	0.253	0.190
5	250	0.864	0.597	0.262	0.258	0.141	0.577	0.471
5	500	1.000	0.669	0.849	0.660	0.465	0.996	0.948
10	100	0.106	0.505	0.066	0.143	0.086	0.216	0.172
10	250	0.186	0.516	0.139	0.196	0.099	0.303	0.276
10	500	0.796	0.548	0.250	0.244	0.140	0.509	0.533

Table A3.(Cont.) Test for no-cointegration, empirical rejection frequency of 5% tests Symmetric EQ- Specification (*ϕ=-0.1, d=9*)

c	*T*	ADF	Engel-Granger	Enders-Granger	Johansen		HW	BVD
					Trace	Max		
3	100	0.302	0.557	0.126	0.144	0.069	0.278	0.260
3	250	0.974	0.631	0.474	0.321	0.166	0.741	0.689
3	500	1.000	0.693	0.998	0.858	0.769	1.000	1.000
5	100	0.171	0.541	0.105	0.147	0.090	0.239	0.228
5	250	0.794	0.574	0.345	0.224	0.113	0.483	0.539
5	500	1.000	0.639	0.920	0.523	0.325	0.983	0.983
10	100	0.098	0.544	0.081	0.153	0.085	0.199	0.184
10	250	0.180	0.536	0.156	0.152	0.081	0.245	0.294
10	500	0.790	0.561	0.346	0.218	0.106	0.481	0.596

Table A3.(Cont.) Test for no-cointegration, empirical rejection frequency of 5% tests Symmetric EQ- Specification (*ϕ=-0.1, d=12*)

c	*T*	ADF	Engel-Granger	Enders-Granger	Johansen		HW	BVD
					Trace	Max		
3	100	0.266	0.558	0.068	0.154	0.080	0.253	0.194
3	250	0.938	0.613	0.342	0.275	0.132	0.677	0.537
3	500	1.000	0.678	0.966	0.740	0.598	0.996	0.986
5	100	0.149	0.573	0.074	0.108	0.055	0.191	0.187
5	250	0.709	0.579	0.232	0.195	0.092	0.393	0.414
5	500	1.000	0.642	0.852	0.463	0.264	0.939	0.936
10	100	0.099	0.535	0.058	0.142	0.082	0.190	0.169
10	250	0.176	0.567	0.123	0.124	0.047	0.233	0.232
10	500	0.716	0.549	0.258	0.205	0.096	0.403	0.498

Table A3.(Cont.) Test for no-cointegration, empirical rejection frequency of 5% tests Symmetric EQ-Specification (*ϕ=-0.6, d=1*)

c	*T*	ADF	Engel-Granger	Enders-Granger	Johansen		HW	BVD
					Trace	Max		
3	100	0.994	0.669	0.377	0.861	0.799	0.998	0.630
3	250	1.000	0.784	0.993	1.000	1.000	1.000	0.999
3	500	1.000	0.910	1.000	1.000	1.000	1.000	1.000
5	100	0.924	0.603	0.175	0.421	0.261	0.831	0.563
5	250	1.000	0.714	0.846	0.961	0.957	0.999	0.987
5	500	1.000	1.000	0.807	1.000	1.000	1.000	1.000
10	100	0.262	0.560	0.061	0.188	0.110	0.339	0.407
10	250	0.835	0.578	0.214	0.285	0.149	0.723	0.825
10	500	0.999	0.658	0.715	0.743	0.610	0.994	0.995

Table A3.(Cont.) Test for no-cointegration, empirical rejection frequency of 5% tests
Symmetric EQ-Specification ($\phi=-0.6$, $d=3$)

c	T	ADF	Engel-Granger	Enders-Granger	Johansen		HW	BVD
					Trace	Max		
3	100	0.992	0.657	0.410	0.812	0.729	0.984	0.637
3	250	1.000	0.772	0.995	1.000	1.000	1.000	0.999
3	500	1.000	0.892	1.000	1.000	1.000	1.000	1.000
5	100	0.928	0.591	0.224	0.427	0.278	0.818	0.484
5	250	0.999	0.689	0.940	0.985	0.975	1.000	0.985
5	500	1.000	0.799	1.000	1.000	1.000	1.000	1.000
10	100	0.319	0.572	0.067	0.189	0.112	0.374	0.281
10	250	0.865	0.584	0.258	0.393	0.236	0.775	0.766
10	500	0.998	0.636	0.818	0.843	0.760	0.995	0.992

Table A3.(Cont.) Test for no-cointegration, empirical rejection frequency of 5% tests
Symmetric EQ-Specification ($\phi=-0.6$, $d=6$)

c	T	ADF	Engel-Granger	Enders-Granger	Johansen		HW	BVD
					Trace	Max		
3	100	0.974	0.615	0.204	0.593	0.464	0.953	0.422
3	250	1.000	0.728	0.967	0.998	0.998	1.000	0.991
3	500	1.000	0.818	1.000	1.000	1.000	1.000	1.000
5	100	0.853	0.586	0.152	0.361	0.231	0.744	0.368
5	250	0.999	0.658	0.706	0.916	0.889	0.998	0.891
5	500	1.000	0.735	1.000	1.000	1.000	1.000	1.000
10	100	0.303	0.537	0.081	0.226	0.129	0.406	0.316
10	250	0.825	0.574	0.172	0.383	0.263	0.736	0.549
10	500	0.999	0.588	0.635	0.796	0.697	0.992	0.946

Table A3.(Cont.) Test for no-cointegration, empirical rejection frequency of 5% tests
Symmetric EQ-Specification ($\phi=-0.6$, $d=9$)

c	T	ADF	Engel-Granger	Enders-Granger	Johansen		HW	BVD
					Trace	Max		
3	100	0.948	0.590	0.250	0.435	0.275	0.874	0.483
3	250	0.714	0.714	0.993	0.979	0.972	1.000	0.996
3	500	1.000	0.787	1.000	1.000	1.000	1.000	1.000
5	100	0.757	0.579	0.192	0.245	0.129	0.636	0.407
5	250	0.998	0.881	0.881	0.822	0.714	0.997	0.952
5	500	1.000	0.720	1.000	0.998	0.998	1.000	1.000
10	100	0.202	0.554	0.092	0.159	0.090	0.341	0.303
10	250	0.778	0.578	0.222	0.277	0.159	0.659	0.531
10	500	0.993	0.615	0.796	0.680	0.556	0.986	0.954

Table A3.(Cont.) Test for no-cointegration, empirical rejection frequency of 5% tests
Symmetric EQ-Specification ($\phi=-0.6, d=12$)

c	*T*	ADF	Engel-Granger	Enders-Granger	Johansen		HW	BVD
					Trace	Max		
3	100	0.911	0.605	0.192	0.378	0.245	0.791	0.397
3	250	0.999	0.678	0.950	0.935	0.893	0.999	0.982
3	500	1.000	0.766	1.000	1.000	0.999	1.000	1.000
5	100	0.667	0.532	0.126	0.218	0.119	0.562	0.372
5	250	0.994	0.667	0.671	0.641	0.495	0.985	0.844
5	500	1.000	0.714	1.000	0.992	0.986	1.000	1.000
10	100	0.198	0.564	0.072	0.173	0.099	0.348	0.305
10	250	0.720	0.571	0.175	0.249	0.142	0.610	0.467
10	500	0.996	0.609	0.697	0.564	0.429	0.977	0.924

Table A3.(Cont.) Test for no-cointegration, empirical rejection frequency of 5% tests
Symmetric EQ-Specification ($\phi=-0.9, d=1$)

c	*T*	ADF	Engel-Granger	Enders-Granger	Johansen		HW	BVD
					Trace	Max		
3	100	0.998	0.682	0.477	0.959	0.937	1.000	0.689
3	250	1.000	0.829	1.000	1.000	1.000	1.000	1.000
3	500	1.000	0.921	1.000	1.000	1.000	1.000	1.000
5	100	0.941	0.619	0.251	0.637	0.521	0.943	0.755
5	250	1.000	0.723	0.946	0.988	0.973	1.000	0.997
5	500	1.000	0.837	1.000	1.000	1.000	1.000	1.000
10	100	0.442	0.530	0.093	0.251	0.171	0.482	0.543
10	250	0.901	0.557	0.345	0.579	0.454	0.884	0.936
10	500	1.000	0.636	0.878	0.935	0.905	0.999	0.996

Table A3.(Cont.) Test for no-cointegration, empirical rejection frequency of 5% tests
Symmetric EQ-Specification ($\phi=-0.9, d=3$)

c	*T*	ADF	Engel-Granger	Enders-Granger	Johansen		HW	BVD
					Trace	Max		
3	100	0.992	0.654	0.497	0.926	0.893	1.000	0.769
3	250	1.000	0.777	1.000	1.000	1.000	1.000	1.000
3	500	1.000	0.898	1.000	1.000	1.000	1.000	1.000
5	100	0.953	0.579	0.244	0.626	0.492	0.936	0.686
5	250	1.000	0.718	0.933	0.996	0.993	1.000	0.993
5	500	1.000	0.797	1.000	1.000	1.000	1.000	1.000
10	100	0.407	0.539	0.071	0.206	0.127	0.481	0.471
10	250	0.900	0.583	0.259	0.550	0.392	0.871	0.899
10	500	0.999	0.627	0.836	0.921	0.888	0.998	0.997

Table A3.(Cont.) Test for no-cointegration, empirical rejection frequency of 5% tests Symmetric Specification (ϕ=-*0.9*, *d*=*6*)

c	*T*	ADF	Engel-Granger	Enders-Granger	Johansen		HW	BVD
					Trace	Max		
3	100	0.978	0.612	0.214	0.765	0.676	0.993	0.524
3	250	1.000	0.741	0.988	1.000	1.000	1.000	0.999
3	500	1.000	0.815	1.000	1.000	1.000	1.000	1.000
5	100	0.883	0.592	0.140	0.482	0.351	0.851	0.501
5	250	0.998	0.657	0.763	0.964	0.931	1.000	0.965
5	500	1.000	0.734	1.000	1.000	1.000	1.000	1.000
10	100	0.332	0.568	0.089	0.219	0.141	0.440	0.356
10	250	0.828	0.563	0.163	0.397	0.278	0.808	0.748
10	500	0.998	0.609	0.684	0.792	0.678	0.992	0.989

Table A3.(Cont.) Test for no-cointegration, empirical rejection frequency of 5% tests Symmetric EQ-Specification (ϕ=-*0.9*, *d*=*9*)

c	*T*	ADF	Engel-Granger	Enders-Granger	Johansen		HW	BVD
					Trace	Max		
3	100	0.954	0.589	0.389	0.547	0.446	0.949	0.677
3	250	1.000	0.717	1.000	0.972	0.960	1.000	1.000
3	500	1.000	0.849	1.000	1.000	1.000	1.000	1.000
5	100	0.801	0.580	0.230	0.298	0.185	0.724	0.588
5	250	1.000	0.682	0.950	0.691	0.546	0.998	0.994
5	500	1.000	0.733	1.000	1.000	0.998	1.000	1.000
10	100	0.256	0.550	0.100	0.188	0.092	0.372	0.424
10	250	0.772	0.569	0.302	0.252	0.150	0.657	0.754
10	500	1.000	0.613	0.895	0.624	0.461	0.980	0.984

Table A3.(Co nt.) Test for no-cointegration, empirical rejection frequency of 5% tests Symmetric EQ-Specification (ϕ=-*0.9*, *d*=*12*)

c	*T*	ADF	Engel-Granger	Enders-Granger	Johansen		HW	BVD
					Trace	Max		
3	100	0.912	0.588	0.263	0.476	0.355	0.878	0.548
3	250	0.999	0.703	0.978	0.873	0.801	0.999	0.995
3	500	1.000	0.790	1.000	1.000	1.000	1.000	1.000
5	100	0.672	0.551	0.149	0.247	0.156	0.646	0.486
5	250	0.992	0.629	0.799	0.552	0.387	0.974	0.956
5	500	1.000	0.701	1.000	0.986	0.981	1.000	1.000
10	100	0.198	0.531	0.076	0.176	0.106	0.314	0.329
10	250	0.722	0.556	0.247	0.243	0.132	0.587	0.670
10	500	0.997	0.595	0.791	0.507	0.313	0.959	0.975

Table A4.Test for no-cointegration, empirical rejection frequency of 5% tests
Asymmetric EQ-Specification (ϕ=-0.1, d=1)

$c^{(1)}$	$c^{(2)}$	T	ADF	Engel-Granger	Enders-Granger	Johansen		HW	BVD
						Trace	Max		
-5	3	100	0.348	0.626	0.079	0.227	0.128	0.418	0.197
-5	3	250	0.951	0.692	0.273	0.456	0.289	0.895	0.594
-5	3	500	1.000	0.785	0.867	0.930	0.898	1.000	0.987
-3	5	100	0.356	0.532	0.084	0.215	0.121	0.411	0.207
-3	5	250	0.957	0.514	0.286	0.423	0.260	0.888	0.593
-3	5	500	1.000	0.544	0.863	0.937	0.895	1.000	0.980
-10	5	100	0.138	0.601	0.053	0.193	0.126	0.317	0.173
-10	5	250	0.336	0.687	0.126	0.248	0.156	0.440	0.396
-10	5	500	0.803	0.777	0.255	0.404	0.247	0.810	0.824
-3	10	100	0.184	0.404	0.062	0.191	0.109	0.317	0.195
-3	10	250	0.327	0.379	0.140	0.263	0.182	0.470	0.450
-3	10	500	0.817	0.292	0.336	0.465	0.290	0.925	0.886

Table A4.(Cont.) Test for no-cointegration, empirical rejection frequency of 5% tests
Asymmetric EQ-Specification (ϕ=-0.1, d=3)

$c^{(1)}$	$c^{(2)}$	T	ADF	Engel-Granger	Enders-Granger	Johansen		HW	BVD
						Trace	Max		
-5	3	100	0.325	0.597	0.087	0.182	0.116	0.340	0.203
-5	3	250	0.965	0.685	0.296	0.380	0.225	0.814	0.564
-5	3	500	1.000	0.766	0.898	0.904	0.853	1.000	0.978
-3	5	100	0.323	0.541	0.072	0.192	0.103	0.329	0.183
-3	5	250	0.957	0.513	0.276	0.380	0.212	0.831	0.548
-3	5	500	0.999	0.548	0.891	0.913	0.855	1.000	0.977
-10	5	100	0.127	0.623	0.072	0.184	0.115	0.290	0.181
-10	5	250	0.370	0.677	0.117	0.229	0.148	0.383	0.301
-10	5	500	0.895	0.745	0.311	0.418	0.251	0.857	0.757
-3	10	100	0.188	0.429	0.060	0.149	0.073	0.287	0.162
-3	10	250	0.382	0.360	0.131	0.235	0.130	0.433	0.341
-3	10	500	0.898	0.328	0.403	0.472	0.291	0.946	0.826

Table A4.(Cont.) Test for no-cointegration, empirical rejection frequency of 5% tests Asymmetric EQ- Specification ($\phi=-0.1$, $d=6$)

$c^{(1)}$	$c^{(2)}$	T	ADF	Engel-Granger	Enders-Granger	Johansen		HW	BVD
						Trace	Max		
-5	3	100	0.266	0.572	0.086	0.168	0.084	0.290	0.216
-5	3	250	0.936	0.673	0.329	0.290	0.155	0.708	0.544
-5	3	500	1.000	0.741	0.937	0.799	0.690	1.000	0.986
-3	5	100	0.266	0.530	0.083	0.178	0.083	0.267	0.204
-3	5	250	0.942	0.566	0.322	0.291	0.157	0.691	0.533
-3	5	500	1.000	0.565	0.943	0.820	0.688	1.000	0.986
-10	5	100	0.127	0.616	0.054	0.160	0.080	0.217	0.153
-10	5	250	0.430	0.650	0.165	0.187	0.112	0.371	0.322
-10	5	500	0.910	0.741	0.426	0.332	0.200	0.764	0.761
-3	10	100	0.169	0.430	0.073	0.161	0.088	0.257	0.179
-3	10	250	0.415	0.417	0.168	0.203	0.106	0.403	0.355
-3	10	500	0.925	0.344	0.557	0.404	0.250	0.914	0.847

Table A4.(Cont.) Test for no-cointegration, empirical rejection frequency of 5% tests Asymmetric EQ- Specification ($\phi=-0.1$, $d=9$)

$c^{(1)}$	$c^{(2)}$	T	ADF	Engel-Granger	Enders-Granger	Johansen		HW	BVD
						Trace	Max		
-5	3	100	0.084	0.594	0.102	0.212	0.120	0.339	0.212
-5	3	250	0.321	0.597	0.165	0.274	0.182	0.470	0.323
-5	3	500	0.908	0.670	0.234	0.389	0.261	0.766	0.561
-3	5	100	0.099	0.479	0.074	0.188	0.098	0.323	0.190
-3	5	250	0.330	0.521	0.161	0.266	0.190	0.462	0.310
-3	5	500	0.916	0.482	0.255	0.406	0.268	0.787	0.577
-10	5	100	0.061	0.585	0.063	0.176	0.105	0.261	0.173
-10	5	250	0.089	0.593	0.139	0.252	0.170	0.398	0.264
-10	5	500	0.318	0.631	0.187	0.294	0.207	0.434	0.385
-3	10	100	0.076	0.450	0.081	0.188	0.106	0.289	0.208
-3	10	250	0.221	0.435	0.165	0.269	0.186	0.440	0.310
-3	10	500	0.346	0.363	0.207	0.304	0.204	0.491	0.431

Table A4.(Cont.) Test for no-cointegration, empirical rejection frequency of 5% tests Asymmetric EQ- Specification (ϕ=-*0.1*, *d*=*12*)

$c^{(1)}$	$c^{(2)}$	*T*	ADF	Engel-Granger	Enders-Granger	Johansen		HW	BVD
						Trace	Max		
-5	3	100	0.088	0.598	0.071	0.162	0.085	0.338	0.173
-5	3	250	0.297	0.605	0.182	0.301	0.195	0.491	0.315
-5	3	500	0.890	0.646	0.276	0.448	0.345	0.772	0.544
-3	5	100	0.082	0.485	0.077	0.180	0.104	0.340	0.182
-3	5	250	0.277	0.501	0.196	0.301	0.198	0.505	0.329
-3	5	500	0.878	0.500	0.298	0.427	0.329	0.768	0.538
-10	5	100	0.065	0.581	0.074	0.156	0.095	0.276	0.170
-10	5	250	0.089	0.603	0.165	0.267	0.182	0.407	0.290
-10	5	500	0.290	0.632	0.253	0.345	0.249	0.491	0.415
-3	10	100	0.085	0.443	0.083	0.166	0.101	0.281	0.186
-3	10	250	0.205	0.440	0.193	0.277	0.184	0.457	0.312
-3	10	500	0.331	0.377	0.256	0.336	0.259	0.489	0.435

Table A4.(Cont.) Test for no-cointegration, empirical rejection frequency of 5% tests Asymmetric EQ-Specification (ϕ=-*0.6*, *d*=*1*)

$c^{(1)}$	$c^{(2)}$	*T*	ADF	Engel-Granger	Enders-Granger	Johansen		HW	BVD
						Trace	Max		
-5	3	100	0.965	0.731	0.251	0.610	0.476	0.977	0.589
-5	3	250	0.999	0.828	0.952	0.998	0.997	1.000	0.998
-5	3	500	1.000	0.998	1.000	0.911	1.000	1.000	1.000
-3	5	100	0.962	0.489	0.269	0.582	0.450	0.966	0.626
-3	5	250	1.000	0.554	0.946	0.995	0.993	1.000	0.998
-3	5	500	1.000	0.615	1.000	1.000	1.000	1.000	1.000
-10	5	100	0.480	0.670	0.098	0.216	0.129	0.532	0.491
-10	5	250	0.928	0.758	0.402	0.569	0.427	0.954	0.924
-10	5	500	1.000	0.872	0.944	0.977	0.968	1.000	1.000
-3	10	100	0.469	0.375	0.104	0.279	0.162	0.593	0.505
-3	10	250	0.903	0.292	0.508	0.734	0.606	0.989	0.953
-3	10	500	0.999	0.199	0.990	0.991	0.987	1.000	1.000

Table A4.(Cont.) Test for no-cointegration, empirical rejection frequency of 5% tests Asymmetric EQ-Specification (ϕ=*-0.6, d=3*)

$c^{(1)}$	$c^{(2)}$	*T*	ADF	Engel-	Enders-	Johansen		HW	BVD
				Granger	Granger	Trace	Max		
-5	3	100	0.964	0.690	0.286	0.609	0.493	0.928	0.533
-5	3	250	1.000	0.813	0.975	0.999	0.998	1.000	0.995
-5	3	500	1.000	0.912	1.000	1.000	1.000	1.000	1.000
-3	5	100	0.962	0.526	0.280	0.603	0.487	0.943	0.529
-3	5	250	1.000	0.615	0.973	0.999	0.999	1.000	0.997
-3	5	500	1.000	0.646	1.000	1.000	1.000	1.000	1.000
-10	5	100	0.525	0.638	0.108	0.225	0.120	0.530	0.348
-10	5	250	0.940	0.748	0.481	0.684	0.581	0.958	0.877
-10	5	500	1.000	0.846	0.982	0.991	0.991	1.000	1.000
-3	10	100	0.534	0.413	0.115	0.338	0.212	0.648	0.387
-3	10	250	0.938	0.338	0.542	0.790	0.701	0.982	0.898
-3	10	500	1.000	0.243	0.990	0.999	0.999	1.000	0.999

Table A4.(Cont.) Test for no-cointegration, empirical rejection frequency of 5% tests Asymmetric EQ-Specification (ϕ=*-0.6, d=6*)

$c^{(1)}$	$c^{(2)}$	*T*	ADF	Engel-	Enders-	Johansen		HW	BVD
				Granger	Granger	Trace	Max		
-5	3	100	0.918	0.619	0.150	0.444	0.306	0.840	0.357
-5	3	250	0.998	0.731	0.867	0.985	0.977	1.000	0.940
-5	3	500	1.000	0.826	1.000	1.000	1.000	1.000	1.000
-3	5	100	0.927	0.554	0.174	0.427	0.291	0.865	0.371
-3	5	250	1.000	0.604	0.849	0.990	0.980	1.000	0.942
-3	5	500	1.000	0.698	1.000	1.000	1.000	1.000	1.000
-10	5	100	0.513	0.614	0.102	0.263	0.186	0.531	0.319
-10	5	250	0.902	0.698	0.300	0.584	0.453	0.910	0.688
-10	5	500	0.999	0.814	0.935	0.966	0.947	1.000	0.995
-3	10	100	0.527	0.459	0.097	0.291	0.191	0.559	0.321
-3	10	250	0.924	0.379	0.365	0.670	0.552	0.965	0.742
-3	10	500	0.999	0.333	0.963	0.986	0.984	1.000	0.994

Table A4.(Cont.) Test for no-cointegration, empirical rejection frequency of 5% tests Asymmetric EQ-Specification ($\phi=-0.6$, $d=9$)

$c^{(1)}$	$c^{(2)}$	T	ADF	Engel-Granger	Enders-Granger	Johansen		HW	BVD
						Trace	Max		
-5	3	100	0.867	0.623	0.188	0.329	0.193	0.774	0.432
-5	3	250	1.000	0.713	0.935	0.910	0.876	0.999	0.976
-5	3	500	1.000	0.815	1.000	1.000	1.000	1.000	1.000
-3	5	100	0.876	0.555	0.216	0.324	0.176	0.772	0.441
-3	5	250	1.000	0.578	0.961	0.922	0.878	1.000	0.988
-3	5	500	1.000	0.658	1.000	1.000	1.000	1.000	1.000
-10	5	100	0.410	0.595	0.096	0.198	0.101	0.428	0.346
-10	5	250	0.915	0.716	0.417	0.417	0.271	0.870	0.728
-10	5	500	1.000	0.792	0.969	0.908	0.853	1.000	0.996
-3	10	100	0.477	0.470	0.131	0.253	0.154	0.509	0.345
-3	10	250	0.920	0.415	0.525	0.543	0.391	0.937	0.795
-3	10	500	0.999	0.362	0.992	0.961	0.943	1.000	1.000

Table A4.(Cont.) Test for no-cointegration, empirical rejection frequency of 5% tests Asymmetric EQ-Specification ($\phi=-0.6$, $d=12$)

$c^{(1)}$	$c^{(2)}$	T	ADF	Engel-Granger	Enders-Granger	Johansen		HW	BVD
						Trace	Max		
-5	3	100	0.787	0.603	0.209	0.382	0.263	0.766	0.480
-5	3	250	1.000	0.683	0.919	0.726	0.584	0.997	0.982
-5	3	500	1.000	0.781	1.000	0.998	0.998	1.000	1.000
-3	5	100	0.807	0.578	0.212	0.352	0.241	0.774	0.506
-3	5	250	0.998	0.646	0.924	0.717	0.614	0.997	0.973
-3	5	500	1.000	0.733	1.000	1.000	0.999	1.000	1.000
-10	5	100	0.399	0.589	0.106	0.218	0.120	0.473	0.412
-10	5	250	0.872	0.680	0.412	0.378	0.240	0.833	0.799
-10	5	500	0.997	0.768	0.980	0.784	0.686	0.998	0.998
-3	10	100	0.561	0.470	0.164	0.295	0.198	0.583	0.460
-3	10	250	0.913	0.517	0.517	0.505	0.375	0.914	0.850
-3	10	500	1.000	0.471	0.986	0.928	0.893	1.000	0.999

Table A4.(Cont.) Test for no-cointegration, empirical rejection frequency of 5% tests Asymmetric EQ-Specification ($\phi=-0.9$, $d=1$)

$c^{(1)}$	$c^{(2)}$	T	ADF	Engel-Granger	Enders-Granger	Johansen		HW	BVD
						Trace	Max		
-5	3	100	0.950	0.774	0.354	0.818	0.757	0.996	0.751
-5	3	250	0.999	0.878	0.991	0.999	1.000	1.000	1.000
-5	3	500	1.000	0.952	1.000	1.000	1.000	1.000	1.000
-3	5	100	0.957	0.441	0.306	0.842	0.773	0.992	0.746
-3	5	250	1.000	0.422	0.989	0.999	0.998	1.000	1.000
-3	5	500	1.000	0.469	1.000	1.000	1.000	1.000	1.000
-10	5	100	0.513	0.712	0.116	0.379	0.256	0.691	0.601
-10	5	250	0.886	0.841	0.474	0.802	0.703	0.988	0.982
-10	5	500	0.995	0.898	0.979	0.995	0.990	1.000	1.000
-3	10	100	0.49	0.357	0.139	0.434	0.309	0.761	0.614
-3	10	250	0.849	0.229	0.621	0.885	0.828	0.996	0.988
-3	10	500	0.982	0.154	0.999	0.998	0.997	1	1

Table A4.(Cont.) Test for no-cointegration, empirical rejection frequency of 5% tests Asymmetric EQ-Specification ($\phi=-0.9$, $d=3$)

$c^{(1)}$	$c^{(2)}$	T	ADF	Engel-Granger	Enders-Granger	Johansen		HW	BVD
						Trace	Max		
-5	3	100	0.960	0.662	0.303	0.768	0.695	0.986	0.700
-5	3	250	1.000	0.829	0.980	0.999	0.999	1.000	1.000
-5	3	500	1.000	0.908	1.000	1.000	1.000	1.000	1.000
-3	5	100	0.963	0.527	0.335	0.754	0.666	0.988	0.714
-3	5	250	1.000	0.594	0.982	1.000	1.000	1.000	0.999
-3	5	500	1.000	0.630	1.000	1.000	1.000	1.000	1.000
-10	5	100	0.587	0.675	0.122	0.345	0.230	0.655	0.538
-10	5	250	0.952	0.770	0.503	0.809	0.732	0.981	0.936
-10	5	500	1.000	0.865	0.988	0.999	0.996	1.000	0.999
-3	10	100	0.592	0.385	0.111	0.382	0.285	0.722	0.480
-3	10	250	0.925	0.296	0.606	0.863	0.819	0.990	0.925
-3	10	500	1.000	0.222	0.997	1.000	0.999	1.000	1.000

Table A4.(Cont.) Test for no-cointegration, empirical rejection frequency of 5% tests Asymmetric EQ-Specification ($\phi=-0.9$, $d=6$)

$c^{(1)}$	$c^{(2)}$	*T*	ADF	Engel-Granger	Enders-Granger	Johansen		HW	BVD
						Trace	Max		
-5	3	100	0.933	0.574	0.176	0.606	0.491	0.943	0.467
-5	3	250	0.999	0.729	0.879	0.992	0.984	1.000	0.982
-5	3	500	1.000	0.828	1.000	1.000	1.000	1.000	1.000
-3	5	100	0.938	0.572	0.182	0.646	0.528	0.96	0.495
-3	5	250	1	0.629	0.88	0.996	0.99	1	0.978
-3	5	500	1	0.736	1	1	1	1	1
-10	5	100	0.541	0.612	0.072	0.31	0.21	0.6	0.343
-10	5	250	0.929	0.725	0.236	0.633	0.501	0.96	0.806
-10	5	500	0.998	0.817	0.916	0.98	0.975	1	0.996
-3	10	100	0.601	0.456	0.114	0.381	0.272	0.695	0.360
-3	10	250	0.905	0.431	0.289	0.733	0.618	0.987	0.797
-3	10	500	0.995	0.356	0.924	0.992	0.983	1.000	0.996

Table A4.(Cont.) Test for no-cointegration, empirical rejection frequency of 5% tests Asymmetric EQ-Specification ($\phi=-0.9$, $d=9$)

$c^{(1)}$	$c^{(2)}$	*T*	ADF	Engel-Granger	Enders-Granger	Johansen		HW	BVD
						Trace	Max		
-5	3	100	0.897	0.609	0.292	0.398	0.275	0.839	0.626
-5	3	250	0.998	0.738	0.971	0.884	0.821	0.998	0.990
-5	3	500	1.000	0.814	1.000	1.000	1.000	1.000	1.000
-3	5	100	0.891	0.541	0.302	0.397	0.276	0.853	0.640
-3	5	250	0.999	0.620	0.981	0.882	0.816	0.999	0.994
-3	5	500	1.000	0.670	1.000	1.000	1.000	1.000	1.000
-10	5	100	0.464	0.609	0.154	0.233	0.137	0.524	0.478
-10	5	250	0.928	0.717	0.567	0.450	0.279	0.926	0.872
-10	5	500	0.999	0.824	0.987	0.905	0.862	1.000	0.997
-3	10	100	0.569	0.474	0.215	0.321	0.208	0.593	0.495
-3	10	250	0.921	0.445	0.639	0.552	0.430	0.951	0.897
-3	10	500	0.999	0.378	0.996	0.960	0.952	1.000	1.000

Table A4.(Cont.) Test for no-cointegration, empirical rejection frequency of 5% tests Asymmetric EQ-Specification ($\phi=-0.9$, $d=12$)

$c^{(1)}$	$c^{(2)}$	T	ADF	Engel-Granger	Enders-Granger	Johansen		HW	BVD
						Trace	Max		
-5	3	100	0.806	0.598	0.218	0.354	0.231	0.785	0.542
-5	3	250	0.993	0.683	0.910	0.724	0.619	0.996	0.979
-5	3	500	1.000	0.774	1.000	1.000	1.000	1.000	1.000
-3	5	100	0.808	0.560	0.204	0.348	0.222	0.754	0.510
-3	5	250	0.998	0.650	0.922	0.746	0.632	0.992	0.983
-3	5	500	1.000	0.697	1.000	0.998	0.997	1.000	1.000
-10	5	100	0.394	0.605	0.093	0.213	0.117	0.459	0.383
-10	5	250	0.874	0.675	0.390	0.333	0.196	0.823	0.779
-10	5	500	1.000	0.766	0.966	0.785	0.682	0.998	0.998
-3	10	100	0.506	0.523	0.154	0.285	0.174	0.564	0.421
-3	10	250	0.901	0.484	0.522	0.543	0.378	0.907	0.823
-3	10	500	0.997	0.462	0.987	0.920	0.876	0.998	0.999

Table A5.Co integration Test with Larger Power per case
Small Sample

			Small sample T=100			
			Band		EQ	
			1st	2nd	1st	2nd
ϕ=-0.1	d=1	c=3	EngG	HW	ADF	EngG
		c=5	EngG	HW	EngG	HW
		c=10	EngG	HW	EngG	HW
	d=3	c=3	EngG	HW	EngG	ADF
		c=5	EngG	HW	EngG	HW
		c=10	EngG	HW	EngG	HW
	d=6	c=3	EngG	HW	EngG	ADF
		c=5	EngG	HW	EngG	HW
		c=10	EngG	HW	EngG	HW
	d=9	c=3	EngG	HW	EngG	ADF
		c=5	EngG	HW	HW	EngG
		c=10	EngG	HW	EngG	HW
	d=12	c=3	EngG	HW	EngG	ADF
		c=5	EngG	HW	EngG	HW
		c=10	EngG	HW	EngG	HW
ϕ=-0.6	d=1	c=3	ADF	HW	HW	ADF
		c=5	EngG	HW	ADF	HW
		c=10	EngG	HW	EngG	BVD
	d=3	c=3	HW	ADF	HW	ADF
		c=5	EngG	HW	ADF	HW
		c=10	EngG	HW	EngG	BVD
	d=6	c=3	HW	λ_{trace}	ADF	HW
		c=5	HW	λ_{trace}	ADF	HW
		c=10	HW	λ_{trace}	EngG	HW
	d=9	c=3	HW	λ_{trace}	ADF	HW
		c=5	HW	λ_{trace}	ADF	HW
		c=10	HW	λ_{trace}	EngG	HW
	d=12	c=3	HW	λ_{trace}	ADF	HW
		c=5	HW	λ_{trace}	ADF	HW
		c=10	HW	λ_{trace}	EngG	HW

Note: EngGstands for the Enders and Granger test

Table A5.(C ont.) Cointegration Test with Larger Power per case Small Sample

			Small sample T=100			
			Band		EQ	
			1st	2nd	1st	2nd
ϕ=-0.9	d=1	c=3	ADF	HW	HW	ADF
		c=5	EngG	HW	HW	ADF
		c=10	EngG	HW	BVD	EngG
	d=3	c=3	HW	λ_{trace}	HW	ADF
		c=5	HW	λ_{trace}	ADF	HW
		c=10	EngG	HW	EngG	HW
	d=6	c=3	HW	λ_{trace}	HW	ADF
		c=5	HW	λ_{trace}	ADF	HW
		c=10	HW	λ_{trace}	EngG	HW
	d=9	c=3	HW	λ_{trace}	ADF	HW
		c=5	HW	λ_{trace}	ADF	HW
		c=10	HW	λ_{trace}	EngG	BVD
	d=12	c=3	HW	λ_{trace}	ADF	HW
		c=5	HW	λ_{trace}	ADF	HW
		c=10	HW	λ_{trace}	EngG	BVD

Note: EngGstands for the Enders and Granger test

Table A6.Coin tegration Test with Larger Power per case
Moderate Sample

			Moderate sample T=250			
			Band		EQ	
			1st	2nd	1st	2nd
ϕ=-0.1	d=1	c=3	ADF	HW	ADF	HW
		c=5	EngG	HW	ADF	HW
		c=10	EngG	HW	EngG	HW
	d=3	c=3	EngG	ADF	ADF	HW
		c=5	EngG	HW	ADF	HW
		c=10	EngG	HW	EngG	HW
	d=6	c=3	EngG	ADF	ADF	HW
		c=5	EngG	HW	ADF	EngG
		c=10	EngG	HW	EngG	HW
	d=9	c=3	EngG	ADF	ADF	HW
		c=5	EngG	HW	ADF	EngG
		c=10	EngG	HW	ADF	BVD
	d=12	c=3	EngG	ADF	ADF	HW
		c=5	EngG	HW	ADF	EngG
		c=10	EngG	HW	EngG	HW
ϕ=-0.6	d=1	c=3	HW	ADF	ADF,HW, λ_{tr} λ_{max}	
		c=5	ADF	HW	ADF	HW
		c=10	EngG	BVD	ADF	BVD
	d=3	c=3	HW	ADF	ADF,HW, λ_{tr} λ_{max}	
		c=5	HW	EngG	HW	ADF
		c=10	EngG	HW	ADF	HW
	d=6	c=3	HW	λ_{trace}	ADF, HW	
		c=5	HW	λ_{trace}	ADF	HW
		c=10	HW	λ_{trace}	ADF	HW
	d=9	c=3	HW	λ_{trace}	HW	BVD
		c=5	HW	λ_{trace}	ADF	HW
		c=10	HW	λ_{trace}	ADF	HW
	d=12	c=3	HW	λ_{trace}	ADF, HW	
		c=5	HW	λ_{trace}	ADF	HW
		c=10	HW	λ_{trace}	ADF	HW

Note: Note: EngGstands for the Enders and Granger test

Table A6.(C ont.) Cointegration Test with Larger Power per case Moderate Sample

			Moderate sample T=250			
			Band		EQ	
			1st	2nd	1st	2nd
ϕ=-0.9	d=1	c=3	ADF, HW		all	
		c=5	ADF	HW	ADF, HW	
		c=10	EngG	HW	BVD	ADF
	d=3	c=3	HW	EngG	all	
		c=5	HW	EngG	ADF, HW	
		c=10	EngG	HW	ADF	BVD
	d=6	c=3	HW	λ_{trace}	all	
		c=5	HW	λ_{trace}	HW	ADF
		c=10	HW	λ_{trace}	ADF	HW
	d=9	c=3	HW	λ_{trace}	ADF, HW, BVD	
		c=5	HW	λ_{trace}	ADF	HW
		c=10	HW	BVD	ADF	BVD
	d=12	c=3	HW, λ_{tr}, λ_{max}		ADF, HW	
		c=5	HW	λ_{max}	ADF	HW
		c=10	HW	λ_{trace}	ADF	BVD

Note: EngGstands for the Enders and Granger test

Table A7.Co integration Test with Larger Power per case
Large Sample

			Large Sample T=500			
			Band		EQ	
			1st	2nd	1st	2nd
ϕ=-0.1	d=1	c=3	ADF	HW	EngG	HW
		c=5	ADF	HW	HW	ADF
		c=10	EngG	HW	BVD	ADF
	d=3	c=3	ADF	HW	ADF, HW	
		c=5	ADF	HW	ADF, HW	
		c=10	EngG	HW	ADF	EngG
	d=6	c=3	ADF	HW	ADF, EngG	
		c=5	ADF	HW	ADF	HW
		c=10	EngG	HW	ADF	EngG
	d=9	c=3	ADF	HW	ADF, HW, BVD	
		c=5	ADF	HW	ADF, HW, BVD	
		c=10	EngG	HW	ADF	BVD
	d=12	c=3	ADF	HW	ADF	HW
		c=5	ADF	HW	ADF	HW
		c=10	EngG	HW	ADF	EngG
ϕ=-0.6	d=1	c=3	ADF,HW,VBD		all	
		c=5	ADF,HW		all	
		c=10	BVD	EngG	ADF	BVD
	d=3	c=3	HW	ADF	all	
		c=5	BVD	HW	all	
		c=10	BVD	EngG	ADF	BVD
	d=6	c=3	HW	λ_{trace}	all	
		c=5	HW	BVD	all	
		c=10	BVD	HW	ADF	HW
	d=9	c=3	HW	λ_{trace}	all	
		c=5	HW	BVD	ADF, HW, BVD	
		c=10	BVD	HW	ADF	HW
	d=12	c=3	HW	ADF	all	
		c=5	HW	BVD	ADF, EndG, HW, BVD	
		c=10	BVD	HW	ADF	HW

Note: EngGstands for the Enders and Granger test

Table A7.(C ont.) Cointegration Test with Larger Power per case Large Sample

			Large Sample T=500			
			Band		EQ	
			1st	2nd	1st	2nd
ϕ=-0.9	d=1	c=3	all		all	
		c=5	HW	ADF	all	
		c=10	BVD	EngG	ADF	HW
	d=3	c=3	HW	BVD	all	
		c=5	HW	BVD	all	
		c=10	BVD	HW	ADF	HW
	d=6	c=3	HW	λ_{trace}	all	
		c=5	HW	λ_{trace}	all	
		c=10	BVD	HW	ADF	HW
	d=9	c=3	HW	λ_{trace}	all	
		c=5	HW	λ_{trace}	all	
		c=10	BVD	HW	ADF	BVD
	d=12	c=3	all		all	
		c=5	HW	BVD	ADF, EndG, HW, BVD	
		c=10	BVD	HW	ADF	BVD

Note: EngGstands for the Enders and Granger test

Appendix 2. Specification tests.

In this Appendix we report the LR and super-Wald tests not reported in the text.

Table A8.Sup Wald Linearity Test for the Fiscal data.

Country	Ho:	VECM (1)		VECM (1)		2TVECM (2)	
	Ha:	2TVECM (2)		3TVECM (3)		3TVECM (3)	
		F_{12}	P-val.	F_{13}	P-val.	F_{23}	P-val.
Canada	p= 1	7.473793 **	0.026	21.04818 **	0.047	12.34451 *	0.083
(1976:1 - 1995:2)	p= 2	5.52084 **	0.036	16.15494 *	0.089	12.35251 *	0.080
	p= 3	13.40902 **	0.036	15.98626 *	0.061	5.422881 *	0.091
	p= 4	9.416957 *	0.087	22.83986 *	0.085	15.07803 *	0.060
	p= 5	10.95854 **	0.029	21.23818 *	0.056	12.27073 *	0.070
	p=6	9.127878 *	0.091	31.62701 *	0.091	25.41381 **	0.034
France	p= 1	18.19577 **	0.041	25.77316 *	0.052	7.393244 *	0.070
(1970:1 - 2001:2)	p= 2	33.03025 **	0.017	37.30132 **	0.028	3.08455 *	0.061
	p= 3	3.300776 *	0.079	19.32357 *	0.076	16.35527 *	0.073
	p= 4	3.462173 *	0.077	18.86118 *	0.070	16.2986 *	0.075
	p= 5	11.61981 *	0.072	3.090863 *	0.068	351.2488 **	0.031
	p=6	5.558858 *	0.059	16.43162 *	0.062	11.05379 **	0.038
Germany	p= 1	4.69966 **	0.040	12.97328 *	0.090	7.200164 *	0.087
(1963:1 - 1990:4)	p= 2	6.656661 *	0.088	12.12171 *	0.088	18.3098 *	0.053
	p= 3	2.757205 *	0.080	14.51429 *	0.077	13.1448 *	0.052
	p= 4	12.48412 **	0.015	22.12576 *	0.074	9.975225 *	0.052
	p= 5	12.30591 **	0.032	23.86273 **	0.031	863.9232 **	0.014
	p=6	9.673219 *	0.076	16.29439 *	0.071	36.09845 **	0.046
Germany	p= 1	8.269196 **	0.024	21.73958 **	0.043	16.01095 *	0.054
(1991:1 - 2001:1)	p= 2	6.438441 **	0.033	17.8752 *	0.080	14.56913 *	0.087
	p= 3	14.87555 **	0.033	17.77496 *	0.055	4.820417 **	0.042
	p= 4	10.83703 *	0.079	24.72419 *	0.077	15.87714 **	0.049
	p= 5	10.41123 **	0.027	24.78465 *	0.051	15.01614 *	0.064
	p=6	10.7641 *	0.082	34.37505 *	0.082	27.26664 *	0.059

Note: 1) *, **, and *** implies rejection of the relevant null hypothesis at 10%, 5% and 1%significant level, respectively.

Table A8.(Cont.) Sup Wald Linearity Test for the Fiscal data.

Country	Ho:	VECM (1)		VECM (1)		2TVECM (2)	
	Ha:	2TVECM (2)		3TVECM (3)		3TVECM (3)	
		F_{12}	P-val.	F_{13}	P-val.	F_{23}	P-val.
Japan	p= 1	20.93983 **	0.037	27.49578 **	0.048	8.507072 *	0.080
(1957:1 - 1980:2)	p= 2	36.87081 **	0.016	38.31713 **	0.026	5.290145 *	0.066
	p= 3	3.821057 *	0.072	20.87828 *	0.069	19.72322 **	0.047
	p= 4	3.886741 *	0.070	21.46117 *	0.063	18.31473 *	0.073
	p= 5	14.56825 *	0.065	2.267698 *	0.062	387.6584 **	0.023
	p=6	5.578035 *	0.053	18.80675 *	0.056	12.77206 **	0.032
United Kingdom	p= 1	6.106847 **	0.037	15.92771 *	0.081	8.370646 *	0.055
(1957:1 - 1998:1)	p= 2	8.365419 *	0.079	10.3955 *	0.080	17.50611 *	0.067
	p= 3	1.785649 *	0.073	15.11353 *	0.070	15.45424 *	0.061
	p= 4	12.6878 **	0.015	22.81214 *	0.067	9.032343 *	0.064
	p= 5	13.15302 **	0.030	23.39491 **	0.029	954.8694 **	0.031
	p=6	11.63649 *	0.068	16.44576 *	0.064	40.57885 **	0.043
U.S.	p= 1	9.365137 **	0.022	23.95381 **	0.039	15.34912 **	0.042
(1946:1 - 2001:2)	p= 2	6.97918 **	0.031	21.64161 *	0.072	15.71066 *	0.073
	p= 3	16.9636 **	0.031	21.44014 *	0.050	7.073881 *	0.052
	p= 4	10.51618 *	0.071	30.78093 *	0.070	20.44895 *	0.082
	p= 5	12.6007 **	0.025	27.4443 **	0.046	15.67893 *	0.054
	p=6	11.02208 *	0.074	38.52281 *	0.074	30.62303 *	0.051

Note: 1) *, **, and *** implies rejection of the relevant null hypothesis at 10%, 5% and 1%significant level, respectively.

Table A9.*L R* Specification Test for the Fiscal data.

Country	Ho:	Band (4)		EQ (5)		EQ (5)	
	Ha:	3TVECM (3)		3TVECM (3)		Band (4)	
		LR_{34}	P-val.	LR_{53}	P-val.	LR_{54}	P-val.
Canada	p= 1	5.900	0.188	16.642	0.441	10.742	0.893
(1976:1 - 1995:2)	p= 2	4.567	0.317	14.081	0.947	9.514	0.957
	p= 3	11.128	0.316	13.545	0.609	2.418	0.902
	p= 4	7.328	0.929	19.789	0.905	12.629	0.945
	p= 5	8.046	0.228	17.406	0.548	9.360	0.658
	p=6	7.289	0.967	26.893	0.968	19.604	0.475
France	p= 1	15.919	0.368	19.983	0.507	4.064	0.962
(1970:1 - 2001:2)	p= 2	28.270 *	0.079	30.263	0.217	1.993	0.947
	p= 3	1.587	0.834	15.285	0.797	13.698	0.978
	p= 4	1.876	0.808	15.590	0.714	13.827	0.530
	p= 5	10.280	0.743	1.717	0.695	311.613 **	0.019
	p=6	3.264	0.585	12.058	0.624	8.794	0.524
Germany	p= 1	3.986	0.365	9.805	0.957	5.820	0.941
(1963:1 - 1990:4)	p= 2	5.925	0.933	7.798	0.938	13.723	0.874
	p= 3	1.112	0.845	11.312	0.803	10.201	0.578
	p= 4	10.029 *	0.065	16.487	0.768	6.518	0.588
	p= 5	9.982	0.264	17.815	0.257	765.869 ***	0.007
	p=6	8.002	0.787	11.807	0.733	29.810	0.834
Germany	p= 1	9.879	0.473	14.233	0.555	4.353	0.880
(1991:1 - 2001:1)	p= 2	5.157	0.632	16.909	0.621	11.752	0.483
	p= 3	8.228 **	0.028	18.032	0.901	9.803	0.406
	p= 4	30.958 *	0.055	44.900	0.732	13.942	0.970
	p= 5	49.529	0.123	105.107	0.276	468.864 **	0.022
	p=6	3.329	0.826	28.269	0.878	21.941	0.695

Note: 1) *, **, and *** implies rejection of the relevant null hypothesis at 10%, 5% and 1%significant level, respectively.

Table A9.(Co nt.) *LR* Specification Test for the Fiscal data.

Country	Ho:	Band (4)		EQ (5)		EQ (5)	
	Ha:	3TVECM (3)		3TVECM (3)		Band (4)	
		LR_{34}	P-val.	LR_{53}	P-val.	LR_{54}	P-val.
Japan	p= 1	2.435	0.263	35.556 **	0.042	23.121	0.970
(1957:1 - 1980:2)	p= 2	11.757 **	0.043	16.792	0.493	5.035	0.730
	p= 3	11.714	0.698	12.261	0.755	0.547	0.777
	p= 4	1.399	0.717	5.514	0.745	5.885	0.808
	p= 5	44.807	0.369	79.947	0.395	35.141	0.206
	p=6	7.157	0.591	282.684	0.147	299.841 *	0.073
United Kingdom	p= 1	7.315	0.204	21.852	0.431	14.537	0.948
(1957:1 - 1998:1)	p= 2	17.288	0.590	28.699	0.711	11.411	0.671
	p= 3	6.301	0.659	16.412	0.306	10.111	0.435
	p= 4	1.793	0.887	21.252	0.852	19.459	0.555
	p= 5	9.054	0.452	12.813	0.579	701.070 ***	0.004
	p=6	4.938	0.760	20.185	0.145	15.247	0.898
U.S.A.	p= 1	5.629	0.574	6.103	0.820	0.474	0.963
(1946:1 - 2001:2)	p= 2	14.477	0.197	13.968	0.347	0.509	0.983
	p= 3	47.140	0.328	52.330	0.316	5.189	0.666
	p= 4	3.415	0.380	3.274	0.935	26.690	0.943
	p= 5	3.621	0.259	-25.683	0.897	29.169	0.897
	p=6	5.719	0.191	17.274	0.465	11.554	0.221

Note: 1) *, **, and *** implies rejection of the relevant null hypothesis at 10%, 5% and 1%significant level, respectively.

Table A10. Super-Wald Specification Test for the Fiscal data.

Country	Ho:	Band (4)		EQ (5)		EQ (5)	
	Ha:	3TVECM (3)		3TVECM (3)		Band (4)	
		F_{43}	P-val.	F_{53}	P-val.	F_{54}	P-val.
Canada	p= 1	7.420733	0.198	19.91858	0.451	12.46214	0.928
(1976:1 - 1995:2)	p= 2	6.439932	0.327	14.45204	0.957	11.79457	0.990
	p= 3	11.77702	0.326	16.08775	0.619	4.817421	0.919
	p= 4	7.590427	0.939	20.24758	0.915	16.13237	0.982
	p= 5	8.565406	0.238	18.9353	0.558	11.35997	0.707
	p=6	8.774853	0.977	27.37045	0.978	23.18119	0.508
France	p= 1	16.44826	0.378	22.76492	0.517	6.137313	0.977
(1970:1 - 2001:2)	p= 2	28.48099 *	0.089	31.24759	0.227	2.808417	0.970
	p= 3	2.161331	0.844	18.26487	0.807	17.14257	1.022
	p= 4	2.921558	0.818	17.58666	0.724	14.94483	0.541
	p= 5	10.98218	0.753	2.368822	0.705	314.6186 **	0.021
	p=6	4.539799	0.595	12.46027	0.634	10.56292	0.553
Germany	p= 1	4.469434	0.375	13.70105	0.967	6.9534	0.960
(1963:1 - 1990:4)	p= 2	7.123697	0.943	11.29454	0.948	15.64833	0.921
	p= 3	2.75506	0.855	11.68084	0.813	13.53616	0.581
	p= 4	11.51375 *	0.075	18.41167	0.778	10.28443	0.629
	p= 5	11.06919	0.274	19.04221	0.267	766.8947 **	0.040
	p=6	9.245951	0.797	13.72183	0.743	30.70951	0.879
Germany	p= 1	11.02149	0.483	16.92565	0.565	6.151368	0.893
(1991:1 - 2001:1)	p= 2	6.216376	0.642	19.24555	0.631	14.08908	0.489
	p= 3	10.00071 **	0.038	21.30608	0.911	9.937039	0.413
	p= 4	31.39216 *	0.065	48.71276	0.742	15.87662	0.995
	p= 5	50.16902	0.133	105.8801	0.286	472.7004 *	0.061
	p=6	4.102159	0.836	29.52668	0.888	24.80847	0.698

Note: 1) *, **, and *** implies rejection of the relevant null hypothesis at 10%, 5% and 1%significant level, respectively.

Table A10. (Cont.) Super-Wald Specification Test for the Fiscal data.

Country	Ho:	Band (4)		EQ (5)		EQ (5)	
	Ha:	3TVECM (3)		3TVECM (3)		Band (4)	
		F_{43}	P-val.	F_{53}	P-val.	F_{54}	P-val.
Japan	p= 1	2.437439	0.273	38.75966 *	0.052	25.85741	0.971
(1957:1 - 1980:2)	p= 2	13.41128 *	0.053	17.75621	0.503	7.504622	0.770
	p= 3	12.14269	0.708	13.80586	0.765	4.001617	0.810
	p= 4	3.324455	0.727	9.121678	0.755	6.784679	0.829
	p= 5	46.698	0.379	81.56139	0.405	38.59603	0.214
	p=6	7.542187	0.601	285.4291	0.157	303.3313	0.108
United Kingdom	p= 1	9.298496	0.214	22.95434	0.441	18.37207	0.955
(1957:1 - 1998:1)	p= 2	19.16742	0.600	29.92504	0.721	12.4028	0.703
	p= 3	6.973113	0.669	18.20718	0.316	13.00306	0.437
	p= 4	3.122772	0.897	23.87056	0.862	19.93603	0.585
	p= 5	10.09603	0.462	16.74572	0.589	703.1494 **	0.011
	p=6	6.031147	0.770	22.98067	0.155	19.15953	0.919
U.S.A.	p= 1	6.103734	0.584	9.145871	0.830	4.167127	0.977
(1946:1 - 2001:2)	p= 2	14.90019	0.207	17.36022	0.357	3.324444	1.024
	p= 3	48.30477	0.338	53.72686	0.326	7.360315	0.703
	p= 4	3.439236	0.390	5.40109	0.945	29.36237	0.981
	p= 5	3.905712	0.269	-22.6602	0.907	32.69879	0.901
	p=6	7.044898	0.201	19.7048	0.475	15.20328	0.270

Note: 1) *, **, and *** implies rejection of the relevant null hypothesis at 10%, 5% and 1%significant level, respectively.

Table A11. Super- Wald Linearity Test for the Trade data.

Country	Ho:	VECM (1)		VECM (1)		2TVECM (2)	
	Ha:	2TVECM (2)		3TVECM (3)		3TVECM (3)	
		F_{12}	P-val.	F_{13}	P-val.	F_{23}	P-val.
Canada	p= 1	10.428	0.517	20.248	0.624	9.280	0.589
(1976:1 - 1995:2)	p= 2	9.856	0.506	28.782	0.636	11.088	0.805
	p= 3	12.844	0.505	23.802	0.484	5.663	0.697
	p= 4	15.245	0.529	45.019	0.430	21.939	0.476
	p= 5	17.802	0.607	49.459	0.548	25.229	0.426
	p= 6	15.593	0.462	57.879	0.300	26.409	0.299
France	p= 1	10.720	0.232	16.589	0.417	5.470	0.206
(1970:1 - 2001:2)	p= 2	7.237	0.889	28.752	0.393	11.903	0.163
	p= 3	9.677	0.770	45.201	0.447	17.259	0.181
	p= 4	11.656	0.850	50.977	0.276	20.630	0.276
	p= 5	12.244	0.607	59.563	0.700	20.966	0.424
	p= 6	14.706 **	0.037	66.546	0.109	32.182	0.120
Italy	p= 1	15.546	0.311	25.718	0.478	9.291	0.726
(1957:1 - 1980:2)	p= 2	18.459	0.450	33.449	0.582	12.521	0.735
	p= 3	29.241	0.795	61.193	0.746	24.679	0.856
	p= 4	42.648	0.748	77.156	0.753	34.508	0.666
	p= 5	49.695	0.714	98.397	0.602	48.702	0.610
	p= 6	16.936	0.132	64.604	0.271	28.521	0.437

Note: 1) *, **, and *** implies rejection of the relevant null hypothesis at 10%, 5% and 1%significant level, respectively.

Table A11. (Cont.) Super- Wald Linearity Test for the Trade data.

Country	Ho:	VECM (1)		VECM (1)		2TVECM (2)	
	Ha:	2TVECM (2)		3TVECM (3)		3TVECM (3)	
Japan	p= 1	59.078	0.113	71.793	0.186	9.547	0.169
(1957:1 - 1998:1)	p= 2	36.637	0.212	49.715	0.498	10.835	0.517
	p= 3	33.362	0.408	51.626	0.388	15.353	0.541
	p= 4	27.089	0.505	64.776	0.106	25.290	0.143
	p= 5	32.194	0.762	77.355	1.000	27.653 ***	0.001
	p= 6	29.726 ***	0.002	95.163	0.715	28.576	0.104
UK	p= 1	14.513	0.666	16.044	0.507	1.416	0.627
(1991:1 - 2001:1)	p= 2	17.666	0.668	61.835	0.256	26.646	0.514
	p= 3	19.363	0.637	67.781	0.244	30.414	0.510
	p= 4	21.732	0.560	73.640	0.321	31.412	0.656
	p= 5	25.278	0.864	79.007	0.345	35.400	0.449
	p= 6	25.178	0.978	89.649	0.536	42.980	0.598
U.S.A	p= 1	5.235	0.702	11.170	0.502	5.766	0.403
(1957:1 - 1980:2)	p= 2	20.517	0.372	38.122	0.111	5.274	0.442
	p= 3	33.502	0.289	73.168	0.681	22.047	0.820
	p= 4	22.000	0.114	48.799	0.411	13.679	0.697
	p= 5	26.597	0.499	61.585	0.430	19.380	0.444
	p= 6	29.088	0.236	73.333	0.427	25.348	0.452

Note: 1) *, **, and *** implies rejection of the relevant null hypothesis at 10%, 5% and 1%significant level, respectively.